The Foreign Exchange Exposure Model (FOREX) Expansion

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Abstract

In January 2007, the theory and application of the FOREX (FOReign EXchange) risk assessment model was developed and applied to the Assistant Deputy Minister (Materiel) (ADM(Mat)) National Procurement and Capital (equipment) accounts to forecast the worse-case loss in expenditures at a specific confidence level over a certain period of time due to the volatility in foreign currency transactions.

With the success of the original FOREX model, the Assistant Deputy Minister (Finance and Corporate Services) has a requirement to expand the model to include the original two ADM(Mat) accounts, national procurement and capital (equipment), plus eight additional funds that each account for over $10M in foreign currency transactions every year. Unlike the manual approach used in the original study, this study uses the Autobox (Automated Box-Jenkins) application to forecast fund expenditures, while GARCH (Generalized Autoregressive Conditional Heteroskedasticity) models are built to forecast the time-varying volatilities of foreign currency returns. These diverse methodologies are then combined into an overall departmental Value-at-Risk model to determine the maximum expected loss from adverse exchange rate fluctuations over the budget year.

Résumé

En janvier 2007, un modèle d'évaluation du risque de change, le modèle FOREX, a été élaboré puis appliqué au compte de l'approvisionnement national et au compte de capital (biens d'équipement) du sous-ministre adjoint (Matériels) (SMA[Mat]) dans le but de calculer, à l'intérieur d'un intervalle de confiance déterminé, la perte maximale qui pourrait découler de la volatilité des taux de change au cours d'une période donnée.

Compte tenu du succès du modèle FOREX initial, le sous-ministre adjoint (Finances et Services du Ministère) (SMA[Fin SM]) doit maintenant élargir la portée de celui-ci et y inclure, en plus des deux comptes du SMA(Mat), huit autres fonds servant tous à financer des opérations en devises totalisant plus de 10 millions de dollars annuellement. La présente étude ne recourt pas à l'approche manuelle adoptée dans le cadre de la première analyse ; elle fait plutôt appel à l'application Autobox (système de modélisation automatique reposant sur la méthode de Box et Jenkins) pour prévoir les dépenses ainsi qu'aux modèles GARCH (modèles généralisés autorégressifs conditionnellement hétéroscédastiques) pour prévoir la variabilité temporelle du rendement des devises. Ces deux méthodes sont ensuite combinées pour créer un modèle de valeur à risque (VAR) propre au ministère qui permet de déterminer la perte maximale qui pourrait découler des fluctuations défavorables des taux de change au cours de l'année budgétaire.

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Executive summary

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P.E. Desmier; DRDC CORA TM 2009–04; Defence R&D Canada – CORA; February 2009.

Value-at-Risk and the FOREX Methodology: In economics and finance, Value-at-Risk, or VaR, is a risk measure that answers the following question: "What is the loss such that it will only be exceeded $p \times 100\%$ of the time in the next K trading days?", where $Pr(Loss > VaR) = p$. Thus, if the VaR on an asset is \$100 million at a one-month, 95% confidence level, there is only a 5% chance that the value of the asset will drop more than \$100 million over any given month.

In the Department of National Defence (DND), the vast majority of foreign exchange exposure comes from the variance (difference) between the exchange rate existing when obligations are budgeted, (b), and those existing when obligations are liquidated, (p). These differences, when multiplied by the expenditure, (E), are generally absorbed within the local budgets that were used to procure the service or equipment. Therefore, being able to predict the rate variances, ($b-p$), with reasonable accuracy would ensure proper management of public funds by minimizing the effects of adverse currency movements.

The monthly-realized budget variance (V) is therefore defined by

$$V = E \times (b-p)\,. \tag{ES.1}$$

Thus, if we simulate the calculation for the budget variance for each fund and currency at each point in time, the VaR is simply the 5th percentile loss, as we have defined it in this analysis, although any parameter of the distribution could be used, with most financial institutions reporting the VaR at the one-day 95% confidence level .

In this and in previous studies [1, 2], we have developed financial expenditure (E) models through Box-Jenkins mechanisms, albeit now automatically produced through the Autobox application; and, have modelled the conditional variances of the financial return series through the basic Generalized Autoregressive Conditional Heteroskedasticity (GARCH)(1,1) model, where the GARCH weights were specified by maximizing the log-likelihood of the standardized *t(d)* distribution for CAD/USD and CAD/GBP, and the normal distribution for CAD/EUR.

The individual models for expenditures and currencies were then combined into an overall departmental VaR model. Results were then obtained through filtered historical simulation (FHS), which assumes no distributional assumptions but retains the non-parametric nature of the historical price change models by bootstrapping from the set of standardized residuals, which were standardized by the GARCH standard deviation.

Monthly forecasted expenditures were matched to exchange rates every 22 trading days to forecast a monthly variance, V. Simulating for 10,000 sequences of hypothetical daily returns,

distributions were produced for expenditures, exchange rates and variances, and the results were validated through interpolating actual values and seeing how well they fit the distribution medians.

With the success of the original FOREX model, ADM(Fin CS) has a requirement to expand the model to include the two funds (national procurement and capital) analyzed in [1], plus eight additional funds that each account for over $10M in foreign transactions every year. This report documents the analysis and validation of the modelling required to calculate the risk of exposure to foreign exchange volatility.

Results: Table ES.1 gives the DND budget rates (b) for equation (ES.1) in its final form. The variance results per month for four months ahead (relative to March 2008) are given in Table ES.2, partitioned by 5th (VaR), 50th (median) and zeroth (maximum expected loss) percentiles of a distribution of 10,000 sequences of equation (ES.1). For example, using the U.S. dollar (USD) Operational Budgets category, which is an aggregation of three funds: L101 (Operating Expenditures), L501 (Minor Requirement/Construction), and L518 (Vote 5 Infrastructure), Figure ES.1 illustrates the output for CAD/USD forecasted operational budget transactions for April 2008 – July 2008 inclusive. The shaded areas to the left and right of average correspond to the lower and upper 5% of the results respectively. Since we are mainly interested in the VaR, the value at the 5th percentile is reported in the upper portion of Table ES.2. The median (50th percentile) of the distribution, which could be a loss or a gain, is reported in the middle portion of the table. Values close to zero imply a budget rate that is close to the forecasted exchange rate. The maximum expected loss (0th percentile) is reported at the bottom of the table and is reflective of significant differences between the budget rate and the forecasted exchange rate.

Figure ES.1 plots the entire variance distribution for each month and shows that each distribution is skewed left with a long tail that is sparsely populated. Clearly extreme values can be reported as, unlike historical simulation, FHS can forecast large losses even if a large loss was never recorded in the historical data set.

The sharp peaks for April and June are unique to this type of analysis and are reflective of the difference calculation in the variance equation (ES.1) where b, the assigned budget rate, is equal to p, the forecasted exchange rate, i.e., the single peak contain the zeros of the variance equation. Single peaks are not found in the charts for May and July because the budget rates were found to be in the tails of the distribution and not around the median.

Table ES.1: DND forecasted budget rate

Months	USD	GBP	EUR
Apr-08	1.0139	2.0089	1.5972
May-08	0.9994	1.9653	1.5555
Jun-08	1.0125	1.9648	1.5757
Jul-08	1.0243	1.9679	1.5771

Table ES.2: Variance and Value-at-Risk forecasted percentile results for U.S. dollar funds

Months	L101	L501	L518	C503	C113	V511	V510	C001	C107	C160	Op. Budget	Invest. Cash	Other
5th percentile loss (Value-at-Risk)													
Apr-08	-577,654	-61,576	-41,926	-1,986,313	-793,377	-3,330,287	-6,757	-100,786	-17,739	-36,606	-1,005,811	-3,601,783	-189,519
May-08	-2,183,803	-235,185	-66,473	-3,187,971	-1,451,853	-5,550,985	-12,297	-178,959	-43,871	-58,129	-2,433,401	-5,825,957	-310,019
Jun-08	-1,260,627	-144,586	-68,635	-3,248,686	-1,431,951	-5,114,664	-10,516	-146,856	-34,506	-65,020	-1,458,758	-5,665,238	-296,685
Jul-08	-1,578,483	-184,070	-71,543	-3,286,016	-1,376,054	-4,932,777	-9,531	-125,907	-48,768	-63,823	-2,315,365	-5,449,278	-292,813
50th percentile gain/loss													
Apr-08	-56,974	0	-5,617	-184,257	-85,067	-1,314	-34	0	0	-3,625	-140,835	-2,059	-3,438
May-08	-465,289	-38,253	-12,237	-416,500	-239,520	-3,338	-80	0	-300	-7,994	-526,779	-5,871	-12,266
Jun-08	-75,805	-1,351	-6,325	-199,321	-105,506	-1,253	-38	0	0	-4,944	-113,314	-1,942	-2,382
Jul-08	-11,007	0	-1,074	-24,231	-4,559	-51	-6	0	0	-775	-37,852	-55	0
Zeroth percentile (expected maximum loss)													
Apr-08	-3,580,841	-628,555	-229,933	-12,027,102	-5,562,590	-29,202,416	-73,699	-1,279,706	-189,789	-196,084	-4,858,550	-29,202,416	-1,642,376
May-08	-10,448,332	-1,806,681	-651,169	-19,105,450	-7,436,613	-50,534,832	-114,370	-1,722,667	-578,966	-350,066	-12,679,790	-38,352,844	-2,058,718
Jun-08	-9,502,858	-1,218,545	-607,640	-23,071,172	-10,900,461	-90,019,000	-162,865	-2,327,150	-552,348	-385,296	-10,749,651	-55,104,280	-3,390,113
Jul-08	-14,778,071	-1,858,528	-1,064,413	-36,772,400	-9,005,898	-82,586,824	-366,072	-4,249,419	-637,456	-527,719	-22,602,636	-69,301,368	-4,475,938

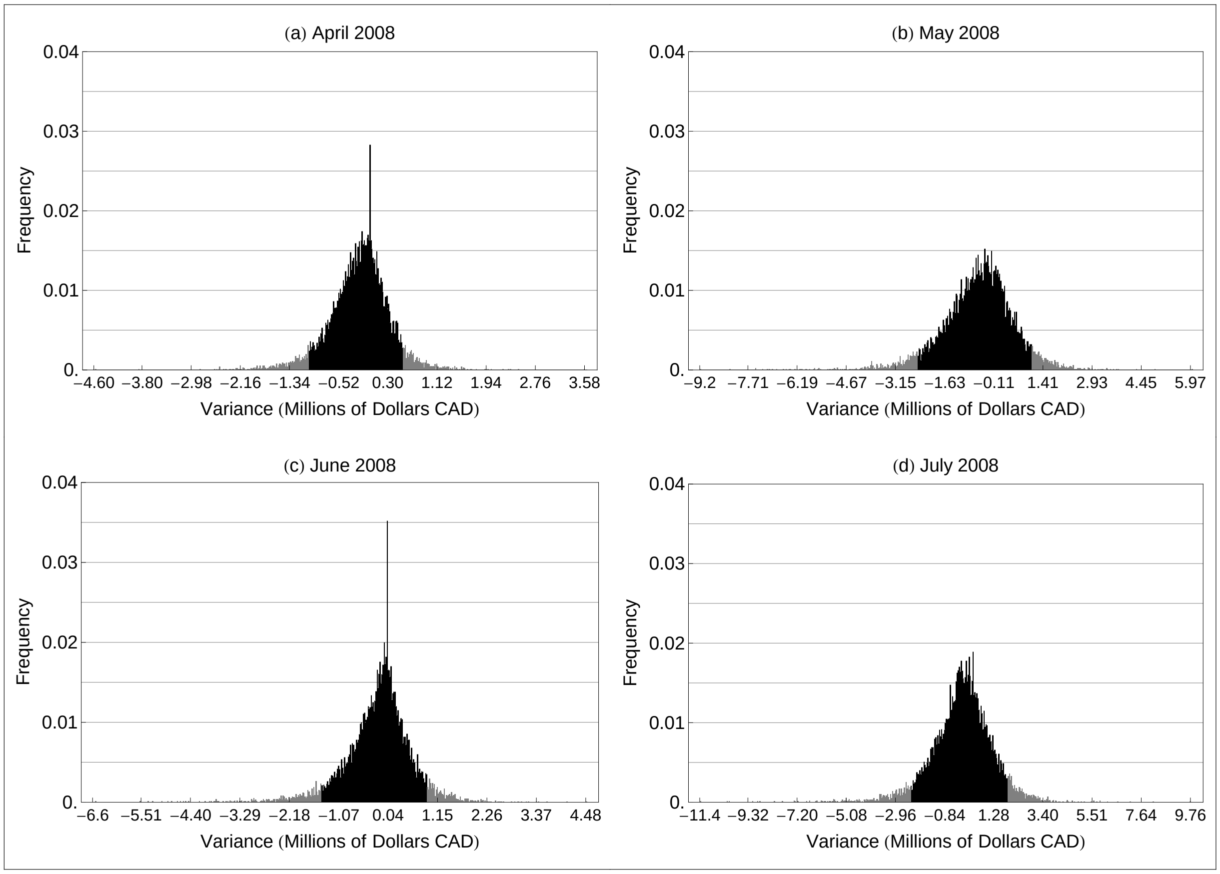

Figure ES.1: Variance forecasted distributions for CAD/USD operational budget fund from April 2008 through July 2008. Shaded areas to left and right of average correspond to the lower and upper 5% of results respectively.

Forecasted Variance Validation: The variance is defined by equation (ES.1) and the Value-at-Risk taken (in this study) as the 5th percentile of the variance distribution. Since we know the actual fund expenditures and exchange rates for April – July 2008, the actual variance could also be calculated. Table ES.3 shows the actual variance for the specified periods as well as where the actuals fall within the VaR distributions (U.S. dollar distributions for the operational budget fund are shown in Figure ES.1).

The results of Table ES.3 provide a useful diagnostic of the VaR models for the funds. There are no observable trends in the percentiles.

The Future: This study further illuminates certain policy implications for functional finance and performance/risk management specialists in the department. In particular, the VCDS Group through the Director Force Planning and Programme Coordination (DFPPC) and ADM(Fin CS) through Director Budget and Director Strategic Finance and Costing will want the capability to adjust corporate budget allocations (quarterly) based on the results of the FOREX model.

Table ES.3: Results of interpolation of actual variance to the forecasted distribution

Fund	April 2008		May 2008		June 2008		July 2008	
	Actual Value	Perc.	Actual Value	Perc.	Actual Value	Perc.	Actual Value	Perc.
L101	69,912	78	218,672	81	-240,978	37	7,201	53
L501	227	80	19,786	82	-12,820	39	252	55
L518	11,870	86	11,153	86	-27,218	24	323	52
C503	19,576	67	66,717	76	-48,465	57	2,013	52
C113	31,394	70	125,751	82	-32,953	56	2,662	53
V511	513,116	89	288	76	0	60	1,098	60
V510	0	65	10	75	-41	49	10	54
C001	0	84	0	88	0	82	0	78
C107	164	84	182	81	-240	35	55	63
C160	3,230	76	473	74	-1,795	57	66	52
Op Budget	82,009	73	249,611	81	-281,016	39	7,776	52
Invest. Cash	513,116	87	299	75	-41	59	1,109	58
Other	3,394	75	655	76	-2,034	51	121	55

Furthermore, these groups should consider adopting the VaR methodology as part of the department's integrated risk management framework for managing the budgetary risk attributed to exposure to foreign currency fluctuations for all acquisitions. Currently there is no tool available to assess the in-year impact of foreign exchange fluctuations on Defence budget allocations. FOREX will offer this capability through an Intranet, Defence Information Network (DIN) based application that is currently under development.

Moreover, should the department decide to seek central government agency concurrence to implement (or pilot) a financial hedging strategy to limit foreign exchange risk (as is the case in the UK), the ability to measure and report exchange rate risk would be fundamental for successful hedging with forward contracts, futures or options. A forward contract would protect the department should the exchange rate depreciate, but on the other hand, the advantage of a favourable exchange rate movement would have to be foregone. Hedging with futures is similar to forwards but is more liquid because it is traded in an organized exchange – the futures market. Currency options provide an insurance against falling below the strike price or the exercise price. However, because options are much more flexible compared to forwards or futures, they are also more expensive.

It remains to be seen if DND's unique requirements could best be served through a combination of options, futures and/or forward contracts. Notwithstanding, this study does illustrate the practical application of the VaR method to arguably the largest department financial risk area, foreign currency exposure, and it is hoped that it will contribute to a better understanding of this risk parameter and how it can be more consistently and accurately measured, reported and ultimately controlled through analysis.

Sommaire

The Foreign Exchange Exposure Model (FOREX) Expansion

P.E. Desmier ; DRDC CORA TM 2009–04 ; R & D pour la défense Canada – CARO ; février 2009.

Valeur à risque et modèle FOREX : Dans les domaines de l'économique et de la finance, la valeur à risque (VAR) est une mesure du risque qui permet de déterminer le montant des pertes qui ne devrait être dépassé que $p \times 100\%$ du temps dans les K prochains jours de bourse, énoncé que l'on peut représenter par l'équation $Pr(perte > VAR) = p$. Ainsi, si la VAR d'un actif, calculée sur un horizon d'un mois et à un seuil de confiance de 95%, équivaut à 100 millions de dollars, cela signifie que la probabilité que la valeur de l'actif accuse une baisse de plus de 100 millions de dollars au cours d'un mois donné n'est que de 5%.

Le risque de change auquel est exposé le ministère de la Défense nationale (MDN) est principalement lié à l'écart (ou la différence) entre le taux de change en vigueur lorsqu'une obligation est budgétée (b) et le taux de change en vigueur lorsque cette même obligation est liquidée (p). Le montant de la différence multipliée par les dépenses (E) est généralement imputé au même budget ayant servi à financer l'achat du bien ou du service en question. Par conséquent, si on était en mesure de prévoir, avec une précision raisonnable, les écarts de taux de change ($b - p$), on pourrait gérer adéquatement les fonds publics en réduisant le plus possible les effets des fluctuations défavorables des cours.

L'écart budgétaire mensuel (V) est donc défini par l'équation suivante :

$$V = E \times (b - p) \qquad \text{(ES.1)}$$

Ainsi, si on simule le calcul de l'écart budgétaire pour chaque fond et pour chaque devise à chaque moment dans le temps, la VAR correspond simplement à la valeur de la perte au 5e percentile, qui est le seuil que nous avons fixé pour la présente analyse quoique n'importe quel paramètre de la distribution pourrait être utilisé. La plupart des institutions financières calculent la VAR à un seuil de confiance de 95% et pour un horizon temporel d'une journée.

Dans le cadre de la présente étude et des analyses antérieures [1, 2], nous avons élaboré des modèles de dépenses (E) à l'aide de la méthode de Box et Jenkins (le processus se fait toutefois automatiquement maintenant grâce à l'application Autobox) puis nous avons modélisé les variances conditionnelles des séries de rendements à l'aide du modèle GARCH(1,1). Les facteurs de pondération du modèle GARCH ont été déterminés en maximisant la fonction de vraisemblance logarithmique des distributions $t(d)$ normalisées établies pour le dollar américain (USD) et la livre sterling (GBP) et de la distribution normale établie pour l'euro (EURO).

Les modèles créés pour les dépenses et les devises ont ensuite été combinés pour former un modèle VAR propre au ministère. Les résultats ont été générés grâce à la simulation historique filtrée, une méthode qui ne repose sur aucune hypothèse de distribution mais qui conserve la nature non paramétrique des modèles de fluctuations historiques des prix en appliquant la

méthode du bootstrap à l'ensemble des résidus normalisés par l'écart type des distributions GARCH.

Les dépenses mensuelles prévues ont été appariées aux taux de change tous les 22 jours de bourse afin de prévoir l'écart budgétaire mensuel (V). Des distributions ont été générées pour les dépenses, les taux de change et les écarts budgétaires sur la base de 10 000 suites de rendements quotidiens hypothétiques, et les résultats ont été validés en interpolant les valeurs réelles dans les distributions et en examinant dans quelle mesure elles se rapprochaient de la médiane.

Compte tenu du succès du modèle FOREX initial, le sous-ministre adjoint (Finances et Services du Ministère) (SMA[Fin SM]) doit maintenant élargir la portée de celui-ci et y inclure, en plus des deux comptes analysés en [1], huit autres fonds servant tous à financer des opérations en devises totalisant plus de 10 millions de dollars annuellement. Le présent rapport porte sur l'analyse et la validation du modèle permettant de calculer le risque associé aux fluctuations des taux de change.

Résultats : Le tableau ES.1 présente les taux budgétés par le MDN (b) qui ont été utilisés pour calculer l'équation (ES.1) dans sa forme finale. Les écarts mensuels calculés pour les mois d'avril 2008 à juillet 2008 (horizon de quatre mois par rapport à mars 2008) sont répertoriés dans le tableau ES.2 et ventilés selon le 5e percentile (VAR), le 50e percentile (médiane) et le percentile 0 (perte maximale prévue) d'une distribution de 10 000 résultats de l'équation (ES.1). Par exemple, la figure ES.1 illustre les distributions des écarts prévus pour les mois d'avril 2008 à juillet 2008 relativement à la catégorie du budget des opérations en dollars américains (USD), qui regroupe en fait trois fonds, soit le compte L101 (dépenses d'exploitation), le compte L501 (besoins mineurs/construction) et le compte L518 (infrastructure - crédit 5). Les zones ombrées à gauche et à droite de la moyenne correspondent aux résultats des première et dernière tranches de 5% de la distribution. Puisque c'est la VAR qui nous intéresse principalement, les valeurs correspondant au 5e percentile figurent dans la section supérieure du tableau ES.2. La section du milieu contient les médianes (50e percentile) des distributions. Celles-ci peuvent représenter un gain ou une perte. Les valeurs près de zéro impliquent que le taux budgété se rapproche du taux de change anticipé. Les pertes maximales prévues (percentile 0) figurent au bas du tableau et font état d'une différence marquée entre le taux budgété et le taux de change prévu.

La figure ES.1 illustre, pour chaque mois, la distribution complète des écarts. On constate que dans les quatre cas, la courbe est désaxée vers la gauche et que la queue de la distribution est longue et contient peu de données. Les valeurs extrêmes peuvent être prises en considération puisque la simulation historique filtrée, contrairement à la simulation historique, permet de prévoir les pertes importantes même si l'ensemble de données historiques sous-jacent n'en contient pas.

Les pics prononcés observés en avril et en juin sont une caractéristique propre à ce genre d'analyse et font état d'une situation où, dans l'équation de l'écart (ES.1), b (le taux budgété) est égal à p (le taux de change prévu). Autrement dit, le pic contient tous les résultats équivalant à 0. Les courbes de mai et juillet ne contiennent pas un tel pic car les taux budgétés se retrouvent dans la queue de la distribution plutôt qu'en périphérie de la médiane.

Tableau ES.1: Taux budgétés par le MDN

Mois	USD	GBP	EUR
Avr. 2008	1,0139	2,0089	1,5972
Mai 2008	0,9994	1,9653	1,5555
Juin 2008	1,0125	1,9648	1,5757
Juil. 2008	1,0243	1,9679	1,5771

Tableau ES.2: Écarts prévus ventilés par percentile, fonds en dollar US

Months	L101	L501	L518	C503	C113	V511	V510	C001	C107	C160	Budget des op.	Investissements	Autres
5th^{e} percentile (valeur à risque)													
Avr. 2008	-577 654	-61 576	-41 926	-1 986 313	-793 377	-3 330 287	-6 757	-100 786	-17 739	-36 606	-1 005 811	-3 601 783	-189 519
Mai 2008	-2 183 803	-235 185	-66 473	-3 187 971	-1 451 853	-5 550 985	-12 297	-178 959	-43 871	-58 129	-2 433 401	-5 825 957	-310 019
Juin 2008	-1 260 627	-144 586	-68 635	-3 248 686	-1 431 951	-5 114 664	-10 516	-146 856	-34 506	-65 020	-1 458 758	-5 665 238	-296 685
Juil. 2008	-1 578 483	-184 070	-71 543	-3 286 016	-1 376 054	-4 932 777	-9 531	-125 907	-48 768	-63 823	-2 315 365	-5 449 278	-292 813
50th^{e} percentile (gain ou perte													
Avr. 2008	-56 974	0	-5 617	-184 257	-85 067	-1 314	-34	0	0	-3 625	-140 835	-2 059	-3 438
Mai 2008	-465 289	-38 253	-12 237	-416 500	-239 520	-3 338	-80	0	-300	-7 994	-526 779	-5 871	-12 266
Juin 2008	-75 805	-1 351	-6 325	-199 321	-105 506	-1 253	-38	0	0	-4 944	-113 314	-1 942	-2 382
Juil. 2008	-11 007	0	-1 074	-24 231	-4 559	-51	-6	0	0	-775	-37 852	-55	0
Percentile 0 (perte maximale prévue)													
Avr. 2008	-3 580 841	-628 555	-229 933	-12 027 102	-5 562 590	-29 202 416	-73 699	-1 279 706	-189 789	-196 084	-4 858 550	-29 202 416	-1 642 376
Mai 2008	-10 448 332	-1 806 681	-651 169	-19 105 450	-7 436 613	-50 534 832	-114 370	-1 722 667	-578 966	-350 066	-12 679 790	-38 352 844	-2 058 718
Juin 2008	-9 502 858	-1 218 545	-607 640	-23 071 172	-10 900 461	-90 019 000	-162 865	-2 327 150	-552 348	-385 296	-10 749 651	-55 104 280	-3 390 113
Juil. 2008	-14 778 071	-1 858 528	-1 064 413	-36 772 400	-9 005 898	-82 586 824	-366 072	-4 249 419	-637 456	-527 719	-22 602 636	-69 301 368	-4 475 938

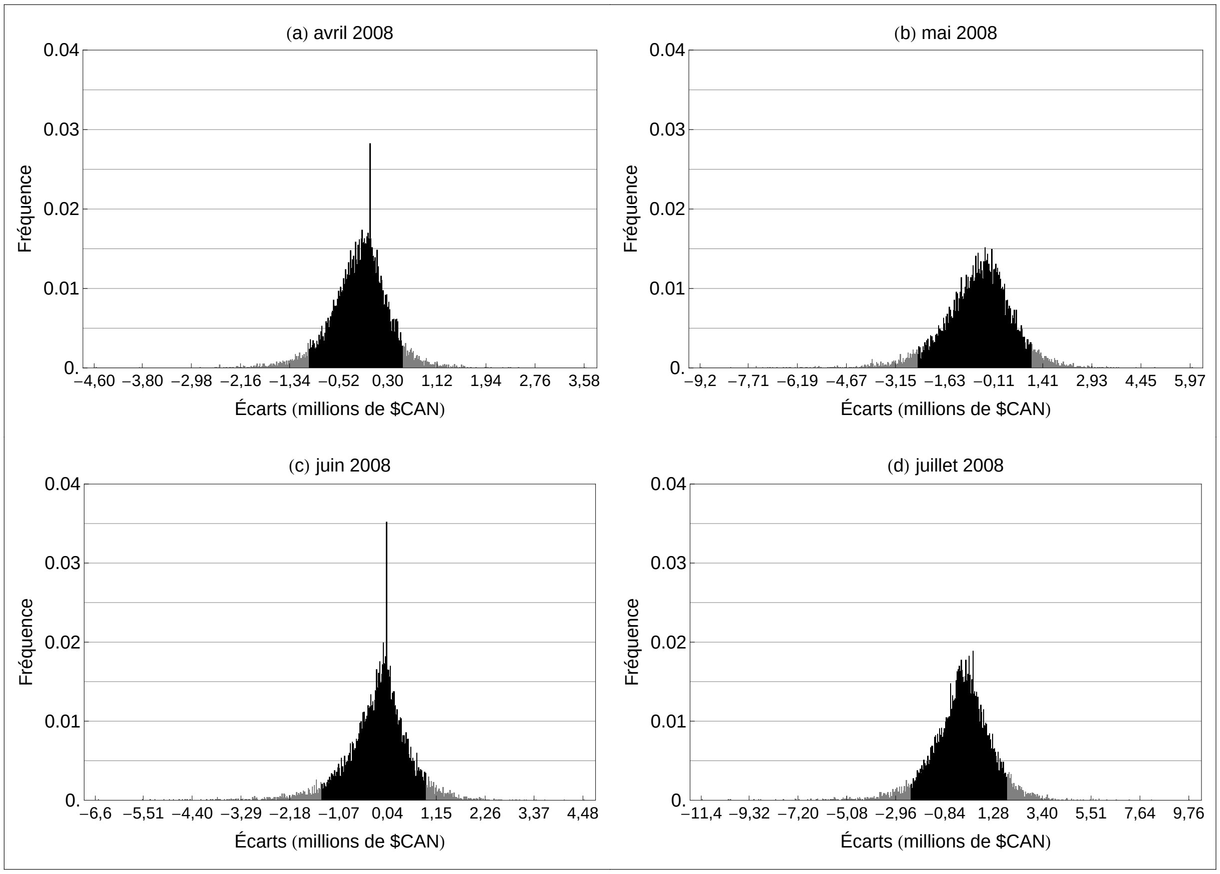

Figure ES.1: Distribution des écarts budgétaires prévus, avril 2008 à juillet 2008, budget des opérations en USD. Les zones ombrées à gauche et à droite de la moyenne correspondent aux résultats des première er dernière tranches de 5% de la distribution.

Validation des écarts prévus : L'écart est représenté par l'équation (ES.1) et la valeur à risque correspond, dans le cadre de la présente étude, au 5e percentile de la distribution. Puisque nous connaissions les dépenses effectuées par le MDN entre avril et juillet 2008 de même que les taux de change en vigueur pendant cette période, l'écart réel pouvait également être calculé. Le tableau ES.3 donne l'écart réel pour les mois examinés de même que la position des valeurs réelles dans les distributions des écarts prévus (les distributions correspondant au budget des opérations en dollars US sont illustrées à la figure ES.1).

Les résultats du tableau ES.3 donnent un aperçu de l'utilité des modèles VAR pour les différents fonds. Aucune tendance particulière ne se dégage des percentiles.

L'avenir : La présente étude met davantage en lumière certaines considérations stratégiques à l'intention des spécialistes du ministère en matière de finances et de gestion du rendement et du risque. En particulier, le groupe du VCEMD, par le truchement du directeur -Planification des Forces et coordination du programme, et le SMA (Fin SM), par l'intermédiaire du directeur - Budget et du directeur - Finances et établissement des coûts (Stratégie), voudront pouvoir

Tableau ES.3: Résultats de l'interpolation des écarts réels dans les distributions des écarts prévus

Fonds	Avril 2008		Mai 2008		Juin 2008		Juillet 2008	
	Valeur réelle	Perc.	Valeur réelle	Perc.	Valeur réelle	Perc.	Valeur réelle	Perc.
L101	69 912	78	218 672	81	-240 978	37	7 201	53
L501	227	80	19 786	82	-12 820	39	252	55
L518	11 870	86	11 153	86	-27 218	24	323	52
C503	19 576	67	66 717	76	-48 465	57	2 013	52
C113	31 394	70	125 751	82	-32 953	56	2 662	53
V511	513 116	89	288	76	0	60	1 098	60
V510	0	65	10	75	-41	49	10	54
C001	0	84	0	88	0	82	0	78
C107	164	84	182	81	-240	35	55	63
C160	3 230	76	473	74	-1 795	57	66	52
Budget des op.	82 009	73	249 611	81	-281 016	39	7 776	52
Investissements	513 116	87	299	75	-41	59	1 109	58
Autres	3 394	75	655	76	-2 034	51	121	55

rajuster les affections budgétaires ministérielles (sur une base trimestrielle) en fonction des résultats du modèle FOREX. En outre, ces groupes devraient envisager d'inclure la méthode VAR dans le cadre de gestion intégrée du risque du ministère, afin de pouvoir gérer, pour toutes les acquisitions, le risque associé aux fluctuations des taux de change. À l'heure actuelle, il n'existe aucun outil permettant d'évaluer l'incidence, en cours d'exercice, des fluctuations des taux de change sur les affectations budgétaires du MDN. Le modèle FOREX offrira cette possibilité par l'intermédiaire d'un réseau d'information de la Défense (RID), qui est en cours d'élaboration et sera intégré à l'intranet.

Par ailleurs, si le ministère devait décider de solliciter l'approbation d'un organisme central en vue de mettre en œuvre (ou de mettre à l'essai) une stratégie de couverture visant à limiter le risque de change (comme c'est le cas au Royaume-Uni), sa capacité à évaluer le risque de change serait indispensable au succès de la stratégie, que celle-ci repose sur des contrats à terme de gré à gré, des contrats à terme standardisés ou sur des contrats d'option. Les contrats à terme de gré à gré protégeraient le ministère si le taux de change devait diminuer. Par contre, le ministère devrait renoncer à tirer profit de toute appréciation des cours. Les contrats à terme standardisés sont une stratégie de couverture semblable aux contrats à terme de gré à gré, mais ils sont plus liquides car négociés sur un marché organisé, à savoir le marché à terme. Les contrats d'option sur devises fournissent quant à eux une protection contre la chute du prix sous le prix d'exercice. Cependant, comme les contrats d'option offrent une plus grande souplesse que les contrats à terme de gré à gré et les contrats à terme standardisés, les prix sont beaucoup plus élevés.

Il reste à savoir si une combinaison de contrats à terme de gré à gré, de contrats à terme standardisés et de contrats d'option conviendrait mieux aux besoins uniques du MDN. Quoi qu'il en soit, cette étude illustre l'application pratique de la méthode VAR au type de risque financier sans doute le plus important au MDN, soit le risque de change. Espérons que cette méthode permettra de mieux comprendre ce risque et de déterminer comment on peut le mesurer et le décrire avec plus de précision et de régularité, pour, en fin de compte, pouvoir le maîtriser.

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1 Introduction

1.1 Background

In January 2007, the mathematical development for the FOREX model - *FOR*eign *EX*change, designed to forecast the foreign exchange risk to the Department of National Defence (DND), was published [1, 2]. As requested by the Director Materiel Group Comptroller (DMG Compt), the study demonstrated the utility of using *Value-at-Risk* (VaR) analysis within the Assistant Deputy Minister (Materiel) (ADM(Mat)) group, for forecasting the potential impact of foreign currency fluctuations of the USD (U.S. dollar), GBP (U.K. pound sterling) and EUR (the euro) exchanges on the ADM(Mat) national procurement (NP) and capital (equipment) accounts, and the application of VaR techniques to determine the maximum expected loss from adverse exchange rate fluctuations over the remaining periods of the budget year. The implementation of foreign exchange exposure risk management, it was decided, would have a definite return on investment for the department. Annually, there is approximately $2.1 billion at risk due to foreign exchange fluctuations. Consequently, being able to forecast losses due to exchange means that procurement/budget managers within capital equipment projects and in-service equipment management teams will ultimately be able to reduce their dependency of holding more money than is necessary for foreign currency losses that may or may not materialize. Therefore, quantifying and managing exchange rate exposure properly means managers can now exercise proper responsiveness to foreign exchange volatility.

Since the prototype FOREX model was developed, there has been a significant level of interest in the modelling expressed by Assistant Deputy Minister (Finance and Corporate Services) (ADM(Fin CS)) staff; therefore, in November 2007 it was decided to modify the scope of the FOREX model to include other components of DND's budget to provide a tool to assess the department's overall exposure to foreign exchange risk [3]. Based on the reporting structure of the Financial Status Report, e.g., see [4], foreign exchange risk would be captured by Defence Service Program (DSP) major expenditure categories for only those funds that contain foreign currency denominated expenditures in excess of $10M. The funds in Table 1 were selected since, in total, they account for 97% of all DND foreign expenditures in the three currencies: USD, GBP and EUR.

Table 1: DSP major expenditure categories and relevant funds

DSP Major Expenditure Categories	Funds		
Operating Budgets[a]	L101	L501	L518
Capital Equipment	C503		
National Procurement	C113		
Investment Cash[b]	V510	V511	
Other[c]	C001	C107	C160

[a]Operating Expenditures (L101), Minor Requirement/Construction (L501), and Vote 5 Infrastructure (L518)
[b]Minor Capital Expenditure Accrual Budgeting (V510) and Capital Expenditure Accrual Budgeting (V511)
[c]Grants & Contributions (C001), Military Cost Moves (C107), and IM/IT Corporate Account (C160)

With the aim of eventually automating the process and creating a web-based departmental application, it became necessary to remove the manual methods of [1, sec. 3] for developing expenditure and currency models, and incorporate an automated process where the time series models for Financial and Managerial Accounting Systems (FMAS) expenditures and foreign currency exchange rates were developed at the outset, but had their coefficients adjusted quarterly as actual data became available. Once a year, it would be necessary to recalculate the models themselves as their structure may have to be adjusted due to radical changes in spending or currency patterns.

While automating currency model updates are relatively straightforward within the main application, such is not the case for FMAS expenditures as they require the modeller to iteratively transform the data, identify trends, seasonal variations or significant points, and run a variety of statistical tests on the model for full validation – and all automated. Neural networks perform best when analyzing monthly or quarterly data, but are technically limited when dealing with daily data as found in most econometric studies. Given their high complexity, they performed no better than traditional automatic Box-Jenkins procedures, which were faster and less resource intensive [5]. In a comparison of neural networks with the Autobox (Automatic Box-Jenkins) application [6] on 50 M-Competition series[1], Kang found Autobox to have superior or equivalent mean absolute percentage error to that for 18 different neural network architectures [11]. Also, in the Tasman-Hoover academic study, Autobox was scientifically ranked best-automated forecasting application [12]. For these reasons and the fact that Autobox is superior to SAS, SPSS and other statistical packages with regard to intervention analysis [13], Autobox was chosen as the application for univariate analysis of the FMAS expenditures.

1.2 Aim

As originally tasked by Director Strategic Finance and Costing (DSFC) [3], the aim of this study is to:

1. develop the FMAS expenditure models for the foreign currency denominated 10 funds listed in Table 1;
2. develop the foreign exchange rate models for the three currencies: USD, GBP, and EUR;
3. combine 1 and 2 into an overall VaR model for DND funds in the three currencies; and,
4. validate the model output against actual data ex ante[2].

[1]Forecasting competitions are designed to compare the forecasting accuracy of different univariate methods on a given collection of time series. The ‘M’-competition series, specifically known as the M-, M2- and M3-competitions, compared 24 methods on 1001 series [7], 24 methods on 29 series [8] and 24 methods on 3003 series [9, 10], respectively.

[2]Ex ante implies an evaluation of the forecast at a later stage when the outcomes are known. Ex post implies an evaluation of the model against a sub-set of the original dataset retained for in-sample forecasts.

1.3 Scope

This report is divided into eight sections. Following the introduction, section 2 describes the data analysis for the two main variables that make up the VaR: Expenditures for the fund categories and foreign exchange rates for the three currencies.

In Section 3, linear (Autobox) models are developed, per currency, for the 10 funds listed in Table 1, and also for the major expenditure categories: Operating Budgets, Investment Cash and Other, for a total of 39 models, i.e.,

$$(\text{10 funds } + 3 \text{ major expenditure categories}) \times \text{ three currencies } = 39 \text{ models}$$

Section 4 presents the conditional GARCH models that accurately model the characteristics of each return series over the 18 year period, 02 April 1990 – 31 March 2008, for USD and GBP; and the nine year period, 04 January 1999 – 31 March 2008, for the EUR. Section 5 builds on the preceding models to construct the overall VaR model — a simulation using the Filtered Historical Simulation (FHS) method of [14]. Results are given in section 6 for forecasted expenditures, currency returns, variance and the 5th percentile VaR. The model is also tested for forecasting performance, ex ante, with four months of data. Section 7 describes the current development of the web-based departmental application, and Section 8 concludes the paper with a discussion on VaR methodology extensions to other areas and a proposal for developing a hedging strategy to limit foreign exchange risk through forward contracts, futures or options.

2 The Data

2.1 What is the Value-at-Risk?

Value-at-Risk, or VaR, is a risk measure that answers the following question: "What is the loss such that it will only be exceeded $p \times 100\%$ of the time in the next K trading days?", where $Pr(Loss > VaR) = p$. As depicted in Figure 1, a VaR calculation is always based on a distribution of possible profits and losses where due to market fluctuations, losses exceeding the VaR amount would occur 5% of the time[3].

While most financial institutions report the VaR at the one-day 95% probability, any parameter of the distribution (e.g., standard deviation of the portfolio return) could be used. Thus VaR can provide a quantitative measure of the downside risk of exposure in all foreign currency transactions.

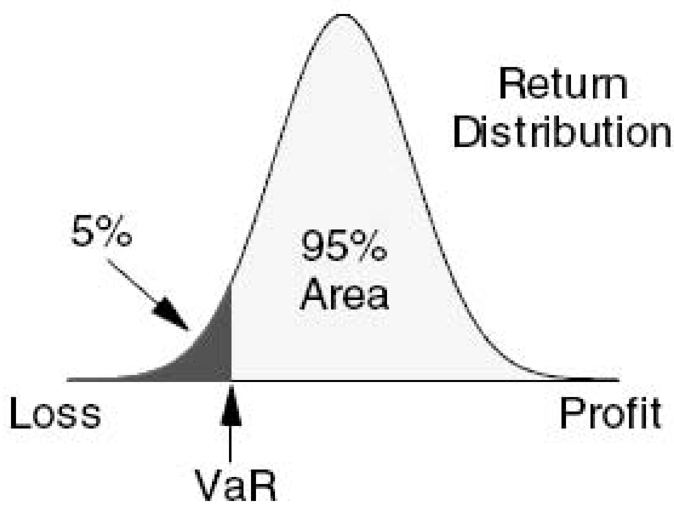

Figure 1: Value-at-Risk (VaR) Example

2.2 The VaR Equation and Budget Variances

Table 1 shows five major expenditure categories with two, NP and capital, consisting of single funds. As stated in [1], in the overall process, the vast majority of foreign exchange exposure comes from the variance (difference) between the exchange rate existing when obligations are budgeted, (b), and those existing when obligations are liquidated, (p). These differences, when multiplied by the expenditure, (E), are generally absorbed within the local budgets that were used to procure the service or equipment. Therefore, being able to predict the rate variances, ($b - p$), with reasonable accuracy would ensure proper management of public funds by minimizing the effects of adverse currency movements. The monthly realized budget variance (V)

[3]Although the return distribution in Figure 1 is shown as normal, in reality it is more peaked about the mean with somewhat fatter tails and best described by the Standardized-t or Generalized Error distributions (see section 4.1 for further details).

is simply the difference between the budget rate (b) and the liquidated rate (p) multiplied by the expenditure (E), i.e.,

$$V = E \times (b - p) \,. \tag{1}$$

Equation 1, in its simplified form, is the basic relationship that defines all VaR calculations for this study. Therefore, if the liquidated exchange rate is greater than the budget rate, a negative variance (loss) is forecasted and a shortfall is presented to the local budget for which funds must be acquired from other sources. Figures 3 – 5 compare the budget rate against the liquidated rate for the USD and the five major expenditure categories: Operating Budgets and Capital (Equipment) Categories (Figure 3), National Procurement and Investment Cash Categories (Figure 4), and the miscellaneous category: Other funds (Figure 5). The USD results are shown as they represent approximately 80% of all foreign exchange transactions from the past ten years (GBP and EUR results are given in Annex A). The expenditure amount and rate at liquidation are proxied by the sum of expenditures at month end and the average monthly rate for each currency.

In Figure 3, capital (equipment) transactions can be, as expected for new equipment purchases, an order of magnitude above operational budget transactions. Consequently, even small differences between the two exchange rates in equation (1) can mean large variances. In the case of the two large negative variance values in March 2001 and March 2002, both are found at the end of the fiscal year (FY) where the summation over periods 12 – 15 can result in seasonal peaks[4].

As far as the exchange rates are concerned, until September 2004 the budget rate was a single, annually forecasted value used per month throughout the FY. Therefore if the actual rate trended up or down, there would be no correction until the next FY. It was unfortunate, for example, that the exchange rate trended upwards at the start of FY 2000/2001 and was not corrected for until 12 months later. From September 2004 to March 2007, the forecasts were monthly and did much better at following the actual rate (the root mean squared error (RMSE) resulting from the annual forecasts was 0.0524) whereas it was 0.0335 for monthly forecasts). From March 2007, in a bid to eliminate volatility, DSFC started generating new forecasts every quarter resulting in an RMSE of 0.0402 (until March 2008 inclusive).

A good example of where even small differences between budgeted and liquidated exchange rates can mean large budget variances is shown in Figure 4 for USD Investment Cash expenditures in July 2007. Two large expenditures of $100M and $485M for the airlift capability project (C-17 acquisition) in the same period, coupled with a difference of 0.036 in the exchange rate, yielded a variance of almost $21M. Annex A contains the rates and Canadian dollar variance on the GBP and EUR liquidated obligations for the five major expenditure categories.

Figure 2 shows the annual realized variances by currency for the five major expenditure categories. Since the USD is the largest contributor to foreign exchange risk, its variance os-

[4]There are 15 periods in FMAS payments for any FY. Periods 1 through 12 represent the months of the standard FY. Periods 13 through 15 are payments captured beyond the FY for which invoices for goods and/or services were submitted prior to 31 March. The latter are normally rolled into period 12, which will tend to “spike” towards an annual distribution at the end of the FY.

cillations will be of greater magnitude. The only exception to this rule is found in the Other category of funds, where several large euro expenditures at end-of-year FY 02/03 and FY 07/08 occurred under a significant difference in exchange rates.

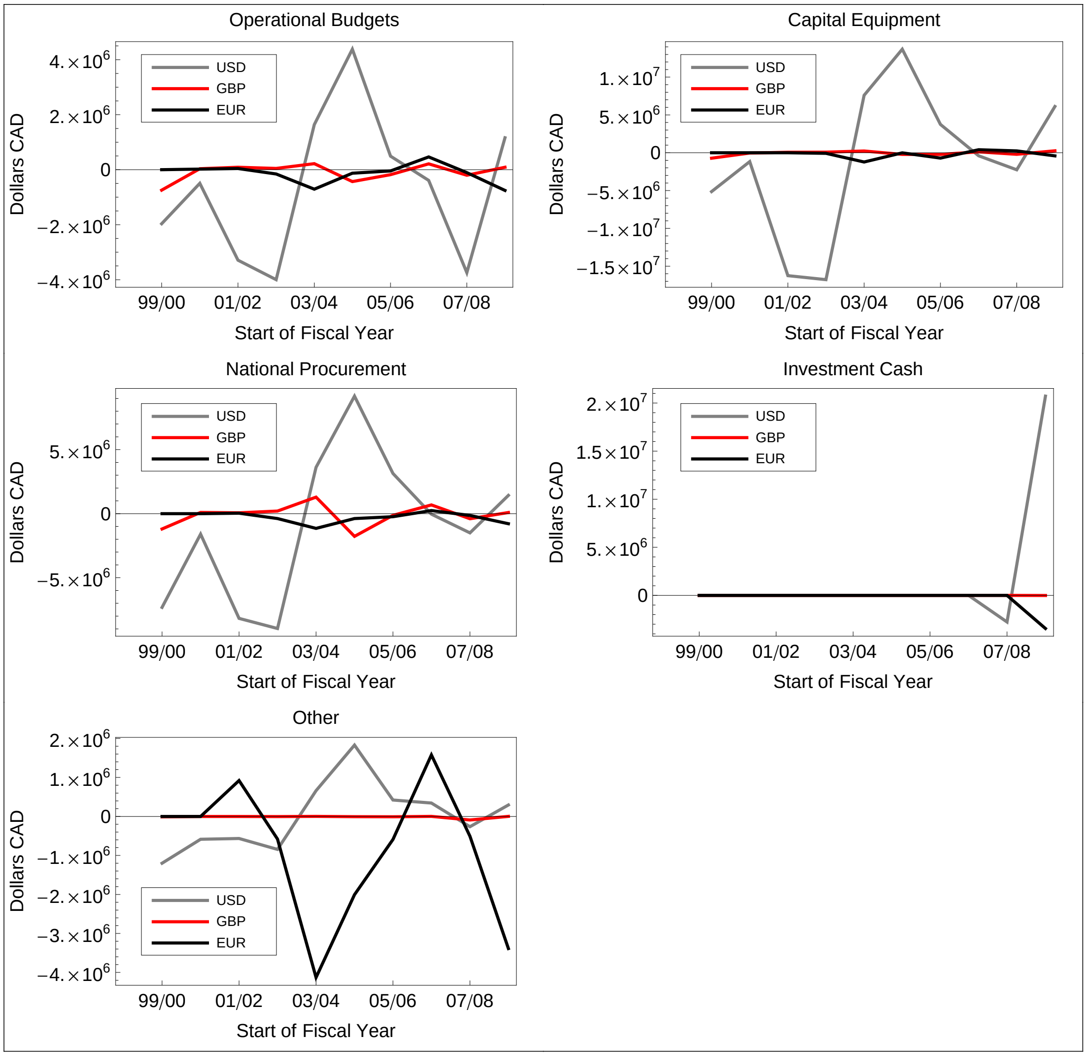

Figure 2: DSP major expenditure category variances for each currency

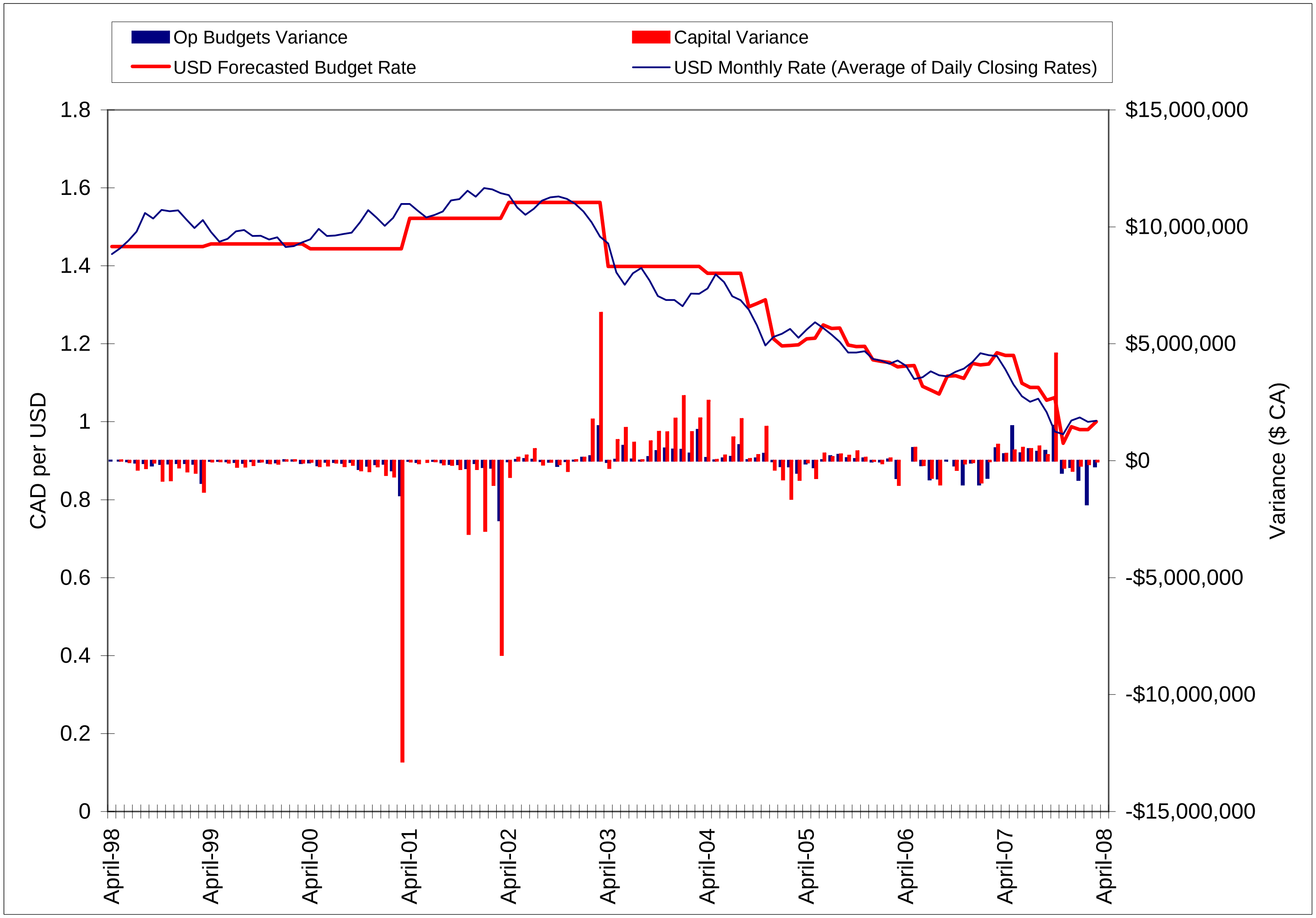

Figure 3: Rates and Canadian dollar variance on U.S. dollar liquidated obligations (Operating Budget and Capital (equipment) categories). Left-hand scale shows exchange rate; Right-hand scale shows variance.

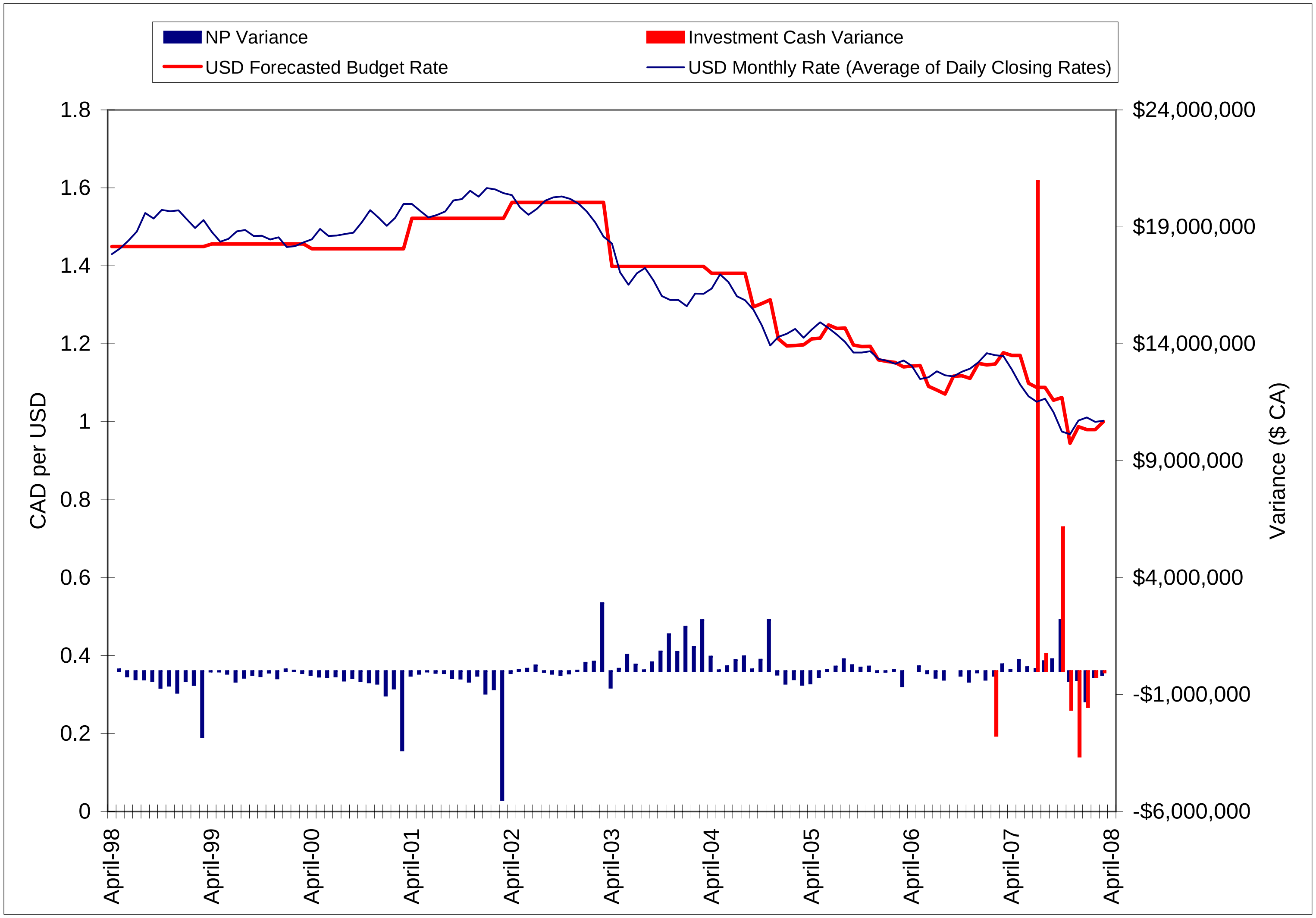

Figure 4: Rates and Canadian dollar variance on U.S. dollar liquidated obligations (National Procurement and Investment Cash categories). Left-hand scale shows exchange rate; Right-hand scale shows variance.

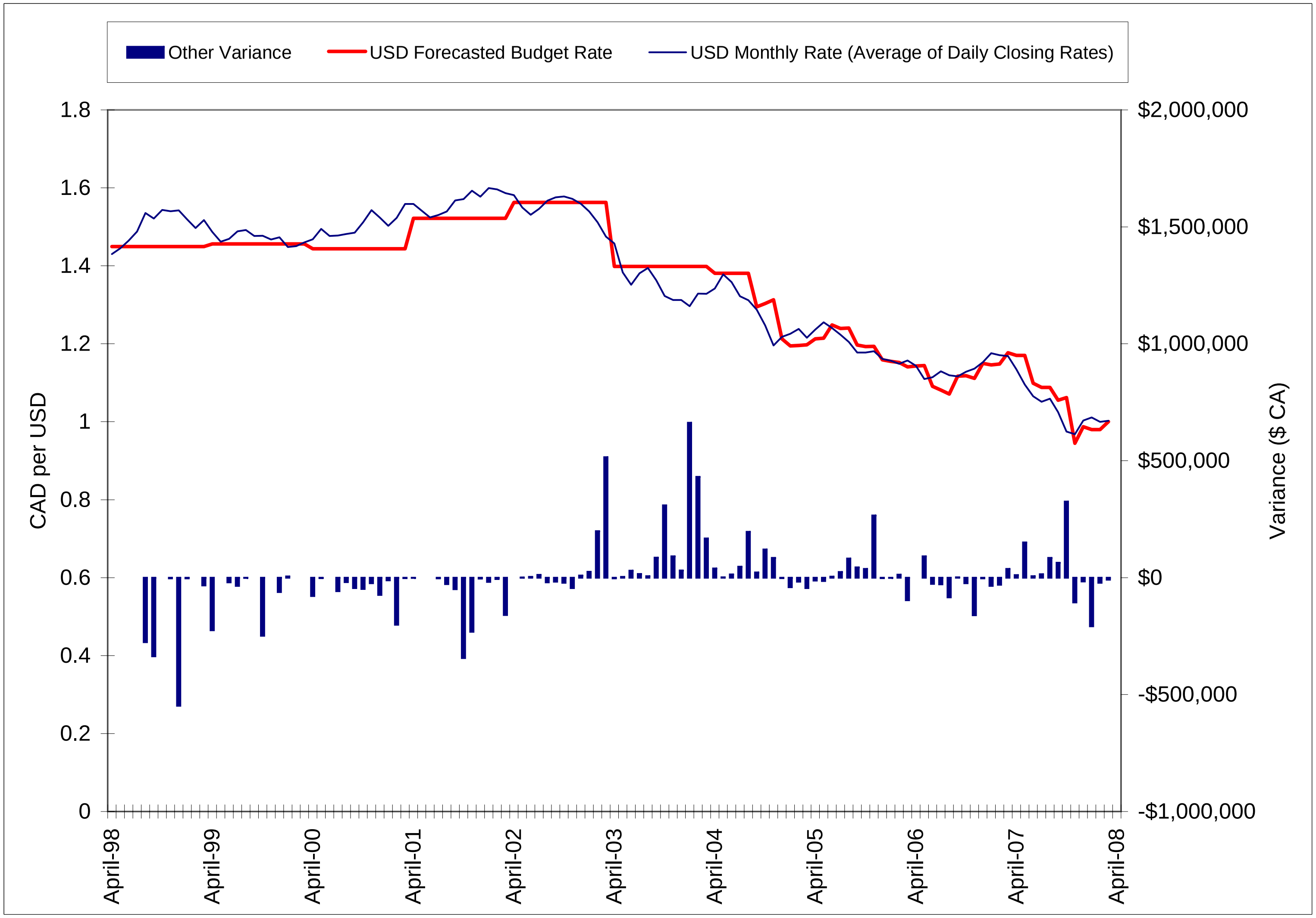

Figure 5: Rates and Canadian dollar variance on U.S. dollar liquidated obligations (Other category). Left-hand scale shows exchange rate; Right-hand scale shows variance.

2.3 DSP Major Expenditure Category Data

Before the FMAS expenditure data can be analyzed, it must be first downloaded from departmental financial web sites and filtered/manipulated according to established rules.

2.3.1 The Revised Rules for Data Filtering

In [1], there were six filtering algorithms designed to analyze, extract and sum dollar amounts for the NP and capital equipment funds. While trying to attain a high-level of accuracy, the algorithms added, it was deemed, an unnecessary high degree of complexity that could be disregarded in the current expansion. Therefore, the following rules were applied [15]:

1. *Extract only KRs*[5]: Reason: Only these Document ID types account for cash outflows.
2. *Use only positive KRs*: Reason: They account for direct purchases.

Therefore, based on these two simple rules, all data was filtered from Director Financial Accounting (DFA)/FMAS extractions under the following fields:

- BFY Budget Fiscal Year;
- AMOUNT Expenditure in Canadian dollars;
- FRNAMT Expenditure in foreign currency;
- CCTR Cost Centres are established to identify responsibility and control costs;
- GL In accounting, GL (General Ledger) accounts belong to one of five types: Assets, Liabilities, Revenue, Expense and either Capital or Surplus;
- FCTR Fund Centre;
- FUND Fund code;
- DT Document Type, e.g., KR (vendor invoice);
- PDATE Posting Date is the date in which the document transaction was to be posted to FMAS;
- FP Financial Period could be 1 (April of current fiscal year) to 15 (June of next fiscal year);
- CK Currency type (USD, GBP, EUR); and,
- CC Capability Component responsible for transaction.

[5] Vendor Invoice (German)

2.3.2 The Funds

Figures 6 – 8 illustrate the distribution of all expenditure data used in this study. Expenses for periods 12-15 were summed under period 12. Even though the euro did not become an official currency until 01 January 1999, it was not forecasted in the DND economic model prior to April 01, 1999. In any case, there were no transactions regarding the euro prior to December 1999.

Inspection of Figures 6 – 8 show strong indication of seasonality, e.g., L501 – all currencies, level shifts, e.g., C503 – USD, and pulses, e.g., V511 – all currencies[6]. Trends, some subjective, in the following funds should be noted (unless otherwise noted, all seasonal pulses are of a 12 month period):

1. L101 — L101 records Vote 1[7] expenditures relating to the acquisition of goods and services [16]. While it may appear that a level shift is required to define the USD model, a better model is obtained by identifying two strong pulses at the end of the series and an autoregressive structure of two polynomials with lags 1 and 12. On the other hand, the GBP model is best defined through a level shift and a series of seasonal pulses. The EUR model relies on a seasonal pulse starting in March 2002 and an autoregressive structure with a polynomial of two parameters with lags one and two. All models visually reflect the rising costs of supplies and services.

2. L501 — L501 records Vote 5 expenditures relating to minor requirements that are less than $5M. Both the USD and GBP record strong seasonal pulses starting in March 2006 and March 2002 respectively. GBP also experiences a negative level shift starting in March 2007. Only the EUR seasonality is defined by a seasonal dummy variable starting in March 2003. The USD and GBP seasonality are defined by a seasonal Autoregressive Integrated Moving Average (ARIMA) structure where the prediction depends on the 12 previous months.

3. L518 — L518 records Vote 5 expenditures relating to infrastructure and environmental activities, and largely for costs pertaining to the construction on various bases, including Afghanistan. While there was very little data available to develop a model, it has been confirmed by Director Financial Arrangements and Support to Operations (DFASO), that the Afghanistan spending patterns for construction should continue for the next two years [17]. Only USD was observed to have a seasonal pulse starting in March 2006 and a minor level shift starting in January 2006.

4. C503 — C503 records Vote 5 capital expenditures relating to major acquisitions of which the U.S. is Canada's major supplier. While there were no established patterns to GBP and EUR spending, the USD experienced a strong level shift starting in March 2001 and a strong seasonal pulse also starting in March 2001, with a reduction in amplitude starting in March 2004.

[6] See section 3.2.1 for full descriptions of the intervention events used in this analysis, i.e., single pulses, seasonal pulses, level shifts and time trends.

5. C113 — C113 records Vote 1 expenditures relating to National Procurement (NP) spending. The NP account usually has a strong seasonal component due to the roll-up of expenditures from periods 12–15 at year-end. In this case, both the USD and GBP show strong seasonal pulses starting in March 2001, while the series for the EUR starts in March 2003. All currencies also have a seasonal ARIMA structure of period 12.

6. V510/V511 — V510/V511 record Vote 5 expenditures and are not/are subject to capitalization. Both V510 and V511 contain the "Investment Funds" as a result of the new accrual budgeting endeavour. While data is initially sparse making model development problematic for both these funds, they are expected to increase dramatically as foreign acquisitions flow through them [18]. At writing, models could not be constructed for V511 (EUR) and V510 (GBP).

7. C001 — C001 records expenditures related to Grants and Contribution payments made under approved terms and conditions. The spending pattern, consisting of zero payments interspersed with actual values, is expected to continue [18]. While there was no discernible spending pattern noted for the USD; for GBP there was a minor seasonal pulse starting in February 2001 and another one starting in June 2002. The EUR exhibited a very strong seasonal pulse starting in March 2004 and an autoregressive structure with a polynomial of two parameters with lags one and two.

8. C107 — C107 records moving expenditures relating to the relocation of military members. For this fund the spending pattern is expected to remain unchanged. Only the USD exhibited a seasonal trend through the ARIMA structure with differencing of period 12.

9. C160 — C160 records Vote 1 expenditures in support of Information Technology (IT) requirements. Only the USD exhibited a seasonal trend with a small seasonal pulse starting in March 2006 and a seasonal ARIMA structure with period 12.

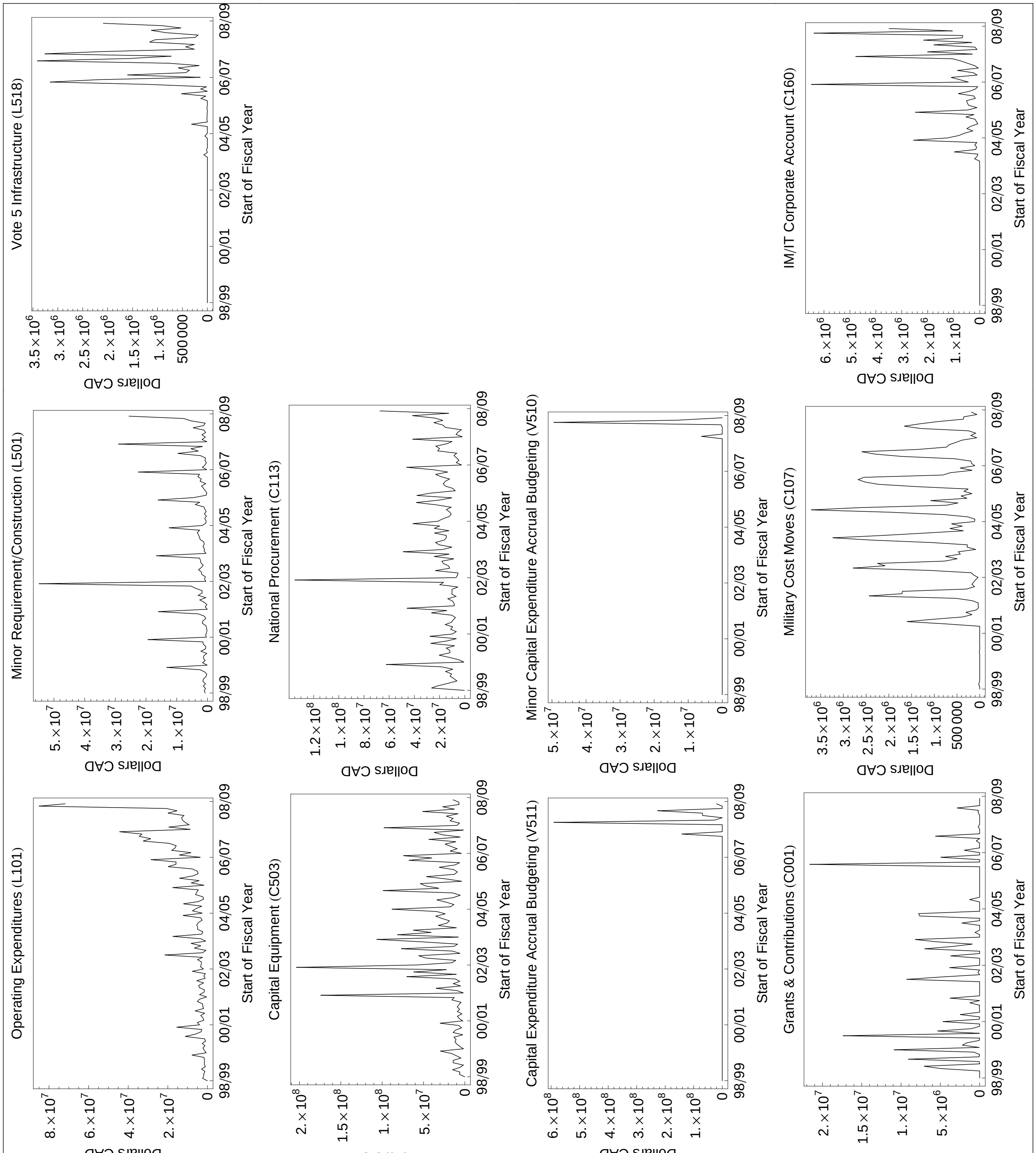

Figure 6: USD liquidated obligations for DSP major expenditure categories

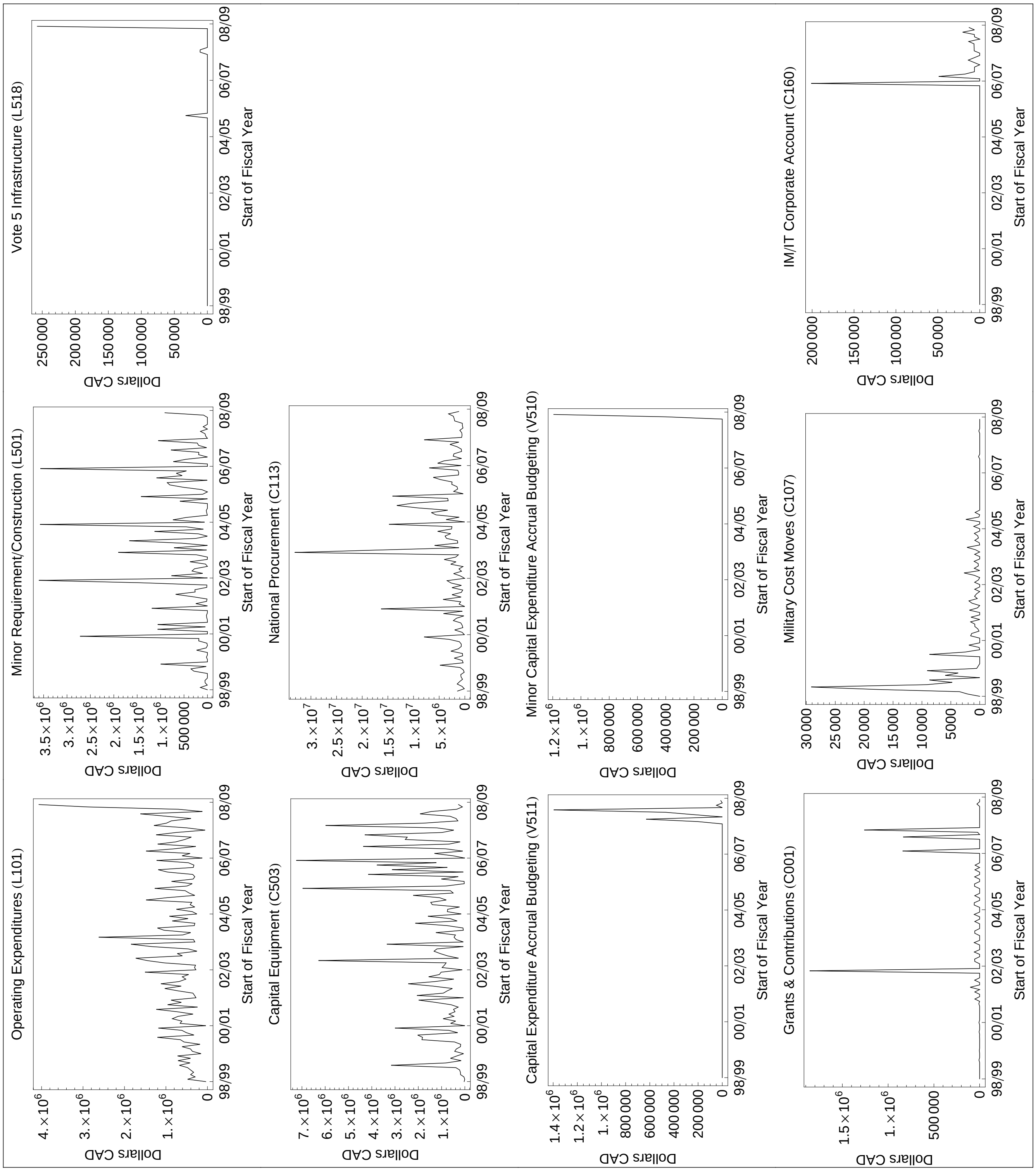

Figure 7: GBP liquidated obligations for DSP major expenditure categories

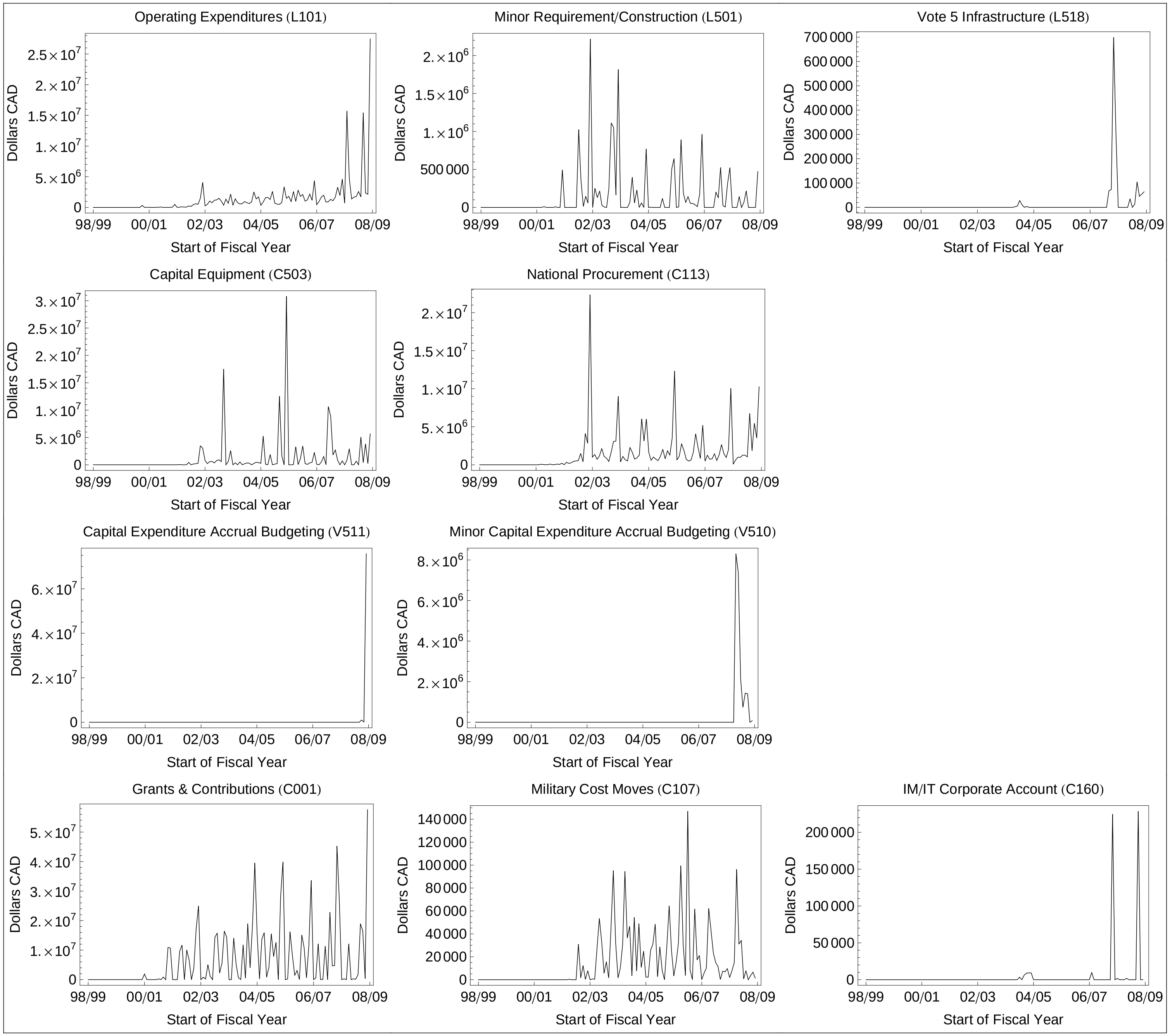

Figure 8: EUR liquidated obligations for DSP major expenditure categories

2.4 The Currencies

Canada has a floating exchange rate, which means there is no set value for the Canadian dollar when compared with any other currency. The exchange rate is affected by supply and demand for Canadian dollars in international exchange markets. If demand exceeds supply, the value of the dollar will go up. If the supply exceeds demand, its value will go down [19]. For VaR applications, closing prices are normally used for assets trading on a local exchange, however, for foreign exchange markets that trade around the clock, the setting of a closing price for instruments trading in different time zones brings a non-synchronicity to the data that must be standardized for it to have any meaning [20].

The Bank of Canada derives its exchange rates from the USD/CAD exchange rate and from indicative wholesale market quotes. The closing rates used in this study are based on official parities or market rates and are updated at about 4:30 p.m. ET on the same business day [21].

Daily closing rates were extracted for the USD and GBP currencies for all trading days from 01 April 1990 through 31 March 2008 (4515 data points). For the EUR, daily closing rates were extracted for all trading days from 01 January 1999 through 31 March 2008 (2320 data points). Figure 9 shows the currency trends over the last seven years. On average, in this period, there were 21 trading days per month $\pm$ 1 day[8]. The trend in the last three years for each currency is downwards. Although conventional wisdom may suggest that the best available model for exchange rate movements is a random walk, it has been argued that traditional economic fundamentals of a country affect to a large extent the equilibrium value of a currency, whose movements are best forecast through more state-of-the-art econometric methods [22].

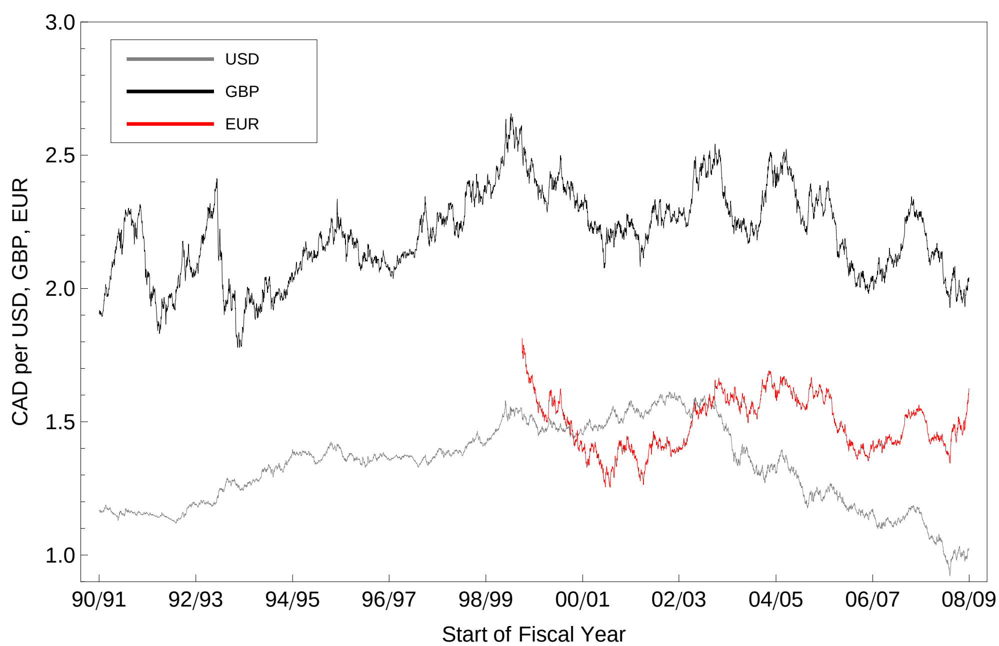

Figure 9: USD, GBP and EUR exchange rates in Canadian dollars

[8]Note, 01 April 2000 and 2001 were non-trading days in Figure 9

3 The Fund Models

The funds are modelled as discrete time series where all transactions during the month are assumed to accumulate at end of month. In this section, a complete analysis is presented of the USD L101 account. It assumes that the reader has some knowledge of time series processes including their prediction and validation.

3.1 Definition and Basic Properties

Let $y_1, \ldots, y_n$ be a stochastic series generated by

$$\phi(B)(y_t - \mu) = \theta(B)\varepsilon_t \;, \tag{2}$$

where μ is the mean parameter, $\phi(B) = 1 - \phi_1 B - \cdots - \phi_p B^p$, $\theta(B) = 1 - \theta_1 B - \cdots - \theta_q B^q$, and ε_t is a sequence of independent, identically distributed (continuous) random variables with mean zero and variance σ^2, i.e., $\varepsilon_t \sim i.i.d.(0, \sigma^2)$[9]. The operator B is the backward shift operator, i.e., $B^k y_t = y_{t-k}$ $(k = 0, \pm 1, \ldots)$, and the polynomials $\phi(z)$ and $\theta(z)$ have their zeros outside the unit circle so that

$$\phi(z) \neq 0 \text{ for } |z| \leq 1 \text{ and } \theta(z) \neq 0 \text{ for } |z| \leq 1 \;. \tag{3}$$

If $\theta_j \neq 0$ for some $j \in \{1, \ldots, q\}$, equation (2) defines a noncausal autoregressive process referred to as purely noncausal when $\phi_1 = \cdots = \phi_p = 0$ [23]. With this definition, it becomes clear that the models developed in [1] were noncausal univariate time series which depended only on current and previous values of the output series, y_t. Causal relationships and intervention variables were not identified largely as a result of the dynamic nature of the data.

3.2 Autobox Modelling

State-of-the-art multivariate modelling procedures ideally combine three types of structures:

$$y_t = \text{Causal} + \text{Memory} + \text{Intervention} \;. \tag{4}$$

Causal events are known events or potential supporting series, which in our case could be macro economic factors such as the Canadian Gross Domestic Product (GDP) growth as it influences defence spending; Memory reflects the history of the input series as lagged variables; and, Interventions reflect omitted causal deterministic series which are empirically defined.

When forecasting with causals, the quality of prediction largely depends upon the quality of the data and the accurate prediction of the future values of the causal variables. This all depends on the accurate identification of causals, quality of data and the timely and accurate input especially regarding interventions.

[9] See [1] (Section 3.1) for further definitions. The notation in equation (2) is slightly different from that of [1]. Here y_t and ε_t were originally defined as X_t and Z_t respectively in equation (2) of [1]

Autobox is an expert system which can be used to model and forecast both univariate and multivariate time series based on Box-Jenkins models. A user specifies an input series and Autobox will automatically correct for omitted variables that have had historical effects, e.g., pulses, seasonal pulses, level shifts and local time trends. Autobox then enhances the forecast model through dummy variables and/or autoregressive memory schemes. Any omitted stochastic series can be identified with an ARIMA structure while any omitted deterministic series can be empirically determined through intervention detection. Autobox then evaluates numerous possible models to find the one that satisfies all necessity tests to guarantee statistically significant coefficients, and all sufficiency tests to ensure that the residuals are a linear combination of zero-mean, uncorrelated random variables or a zero-mean Gaussian white noise process [24].

In developing the fund models, we used no predetermined (causal) input series and instead relied on Autobox to specify an accurate memory structure through lagging the output variables (autoregressive model components), i.e., $y_{t-1}, y_{t-2}, \ldots$, and a set of dummy variables with correct pulses, seasonal pulses, level shifts and spline time trends. In the case of the former, seasonality could also be specified by a seasonal ARIMA memory structure where the forecast could be specified through differencing given a period of 12 months, e.g., $(1-B^{12})$ or autoregressive polynomials $(1-\phi B^{12})$.

3.2.1 Interventions

As already stated, Autobox was the application of choice for the linear evaluation of expenditures largely due to its superior application of intervention analysis and outlier detection on the fund data. Intervention events are known events that can be single pulses whose impact is transitory, reoccurring seasonal pulses, level shifts which reflect sudden changes in the mean, or time trends which can best be described by simple linear models. Well-known and successful examples of intervention analysis are Box and Tiao's study where they developed the basic intervention analysis methodology and applied it to air pollution control and economic policies [25], and Montgomery and Weatherby's impact of the Arab oil embargo [26].

The four types of intervention events are:

- **Pulse** A pulse is a one-time event that needs to be accounted for in order to properly identify the model. If we let x_t define the intervention or dummy variable representation, there are only two values that x_t can take: 0 or 1. For example, for the fund USD L101 consisting of 120 observations, Autobox detected an unusually high value in October 2002 (point 55)[10]. Therefore, if x_t represents a pulse at time period 55, its representation is
$$x_t = \overbrace{0, 0, 0, \ldots, 0, 0}^{\text{54 values}}, 1, \overbrace{0, 0, 0, \ldots, 0, 0}^{\text{65 values}} \ . \tag{5}$$
- **Seasonal Pulse** Seasonal events are defined via a complete or partial set of seasonal dummy variables reflecting a fixed response based upon the specified period. For ex-

[10]Like most single pulses found in this study, the pulse at point 55 is not the result of one single FMAS expenditure, but the sum of a large number of values (411 in this case) of which two are exceedingly high, i.e., \$7.36 M and \$7.99 M both for United States Navy (USN)/CF foreign exchange adjustment reconciliation.

ample, for the fund USD L518 consisting of 44 observations and a period of 12 months, Autobox detected an unusually high set of values 12 months apart starting in March 2006 (datapoint 20). Therefore, if x_t represents a seasonal pulse starting at time period 20, its representation is

$$x_t = \overbrace{0,0,0,\ldots,0,0}^{19\text{ values}},1,\overbrace{0,0,0,\ldots,0,0}^{11\text{ values}},1,\overbrace{0,0,0,\ldots,0,0}^{11\text{ values}},1,\underbrace{\overbrace{0,0,0,\ldots,0,0}^{11\text{ values}},1,0,\ldots}_{\text{forecast}}, \quad (6)$$

where after 44 values, the seasonal pulse is used in the forecast.

- **Level Shift** Level shifts are defined by differences in the means for sets of values in the same series. For example, for the fund USD C503 consisting of 120 observations and a period of 12 months, Autobox detected a significant difference between the means of the first 35 values and the last 85 values, implying a level shift starting March 2001 (datapoint 36). Therefore, if x_t represents a level shift starting at time period 36, its representation is

$$x_t = \overbrace{0,0,0,\ldots,0,0}^{35\text{ values}},\overbrace{1,1,1,\ldots,1,1}^{85\text{ values}},\underbrace{1,1,1,\ldots,1,1,\ldots}_{\text{forecast}}, \quad (7)$$

where after 120 values, the level shift is used in the forecast.

- **Time Trend** Time trends reflect changes in slopes. In time series they require identification of the break points and then estimation of the local trend. It often happens that a time series appears to have a trend, but is not. If the trend is not convincing, Autobox will not develop the model nor forecast the series based on a trend. Such is the case for all the funds in this study.

3.3 A Model for the USD L501 Fund

USD L501 is an interesting series that highlights many of the points discussed in the previous section. The series is of length 120 with sample mean and variance \$3.359M and 4.80×10^{13} respectively. All values are positive and none are zero.

Figure 10 shows how Autobox has defined the structure of the series and has adjusted the values to account for seasonal and one-time events. These have been highlighted as either a seasonal pulse (red “S”) or a single pulse (red “P”). The unadjusted series is found where no “P” or “S” is found. Therefore, the first 36 points, including the three peaks at March 1999, March 2000 and March 2001, are acceptable points for this series and clearly define a monthly seasonality which still needs to be modelled. All points viewed by Autobox as pulses therefore need to be modelled as increments or reductions on the final series.

For example, Autobox found eight single, non-repeatable pulses and one seasonal, repeatable pulse. Point 48 (March 2002) is the largest pulse found with a magnitude of 54.8977×10^6 or, as specified by Autobox, an increment of 37.5181×10^6 over the final series. Similarly, point 72 (March 2004) is smaller than the average March peak, and the final series will need to be reduced accordingly. The seasonal pulse identified at point 96 (March 2006) with increment 6.745×10^6 over the final series, will have this increment added to every 12 points (March) thereafter. This means that point 108 (March 2007), which is already defined as a pulse, will be made up of the value for the final series, plus the seasonal increment from point 96, plus the single pulse increment, 5.107×10^6, to make up the final magnitude of this point.

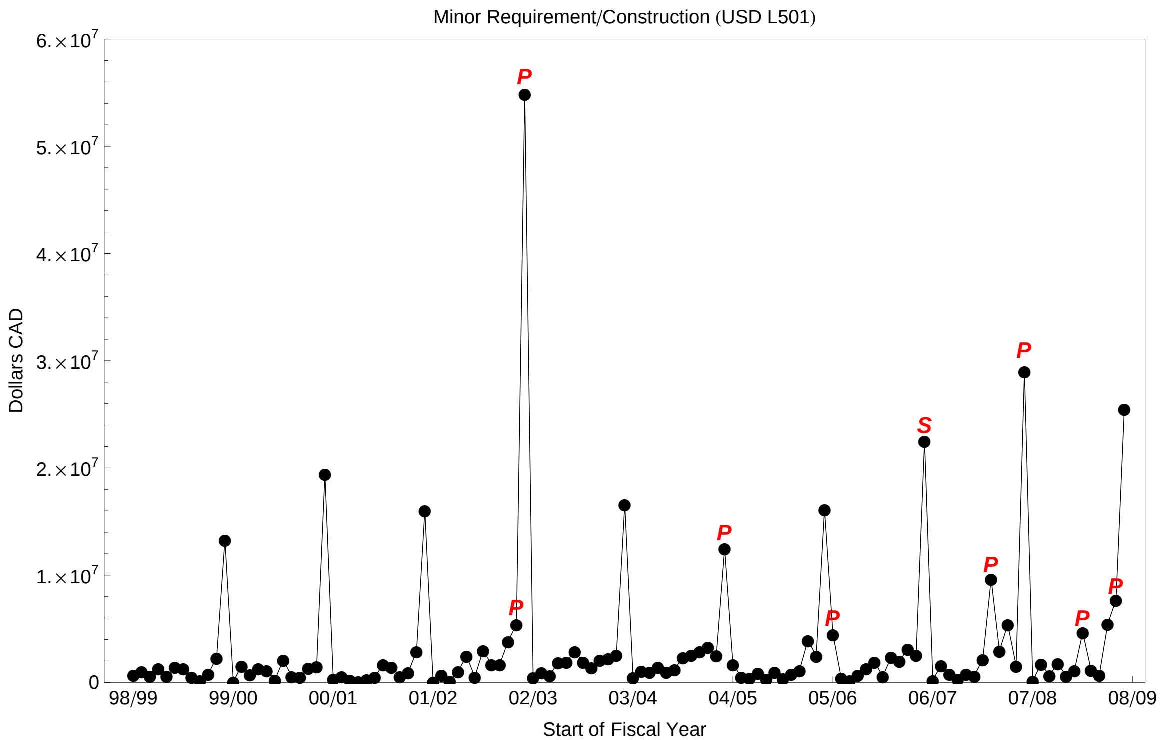

Figure 10: USD L501 fund from 01 April 1998 – 31 March 2008; P = single pulse, S = seasonal pulse

For ease of analysis, all values were divided by 10^6 prior to model development. In the form of equation (2), Autobox generated the following model:

$$y_t = \mu + \sum_{i=1}^{9} c_i x_{t,i} + \frac{\varepsilon_t}{[1-\phi_{12}B^{12}][1+\phi'_{12}B^{12}]}\,, \tag{8}$$

where ϕ_{12} and ϕ'_{12} are differing autoregressive coefficients of order 12. Rearranging equation

(8) with substitutions yields

$$
\begin{aligned}
(1-\phi_{12}B^{12}-\phi_{24}B^{24})(y_t-\mu-\sum_{i=1}^{9}c_i x_{t,i}) &= \varepsilon_t \ , \\
(1-0.525B^{12}-0.471B^{24})(y_t-25.484-\sum_{i=1}^{9}c_i x_{t,i}) &= \varepsilon_t \ , \\
y_t-0.525y_{t-12}-0.471y_{t-24}-0.102-\sum_{i=1}^{9}c_i[1-0.525B^{12}-0.471B^{24}]x_{t,i} &= \varepsilon_t \ . \qquad (9)
\end{aligned}
$$

On the left-hand side of equation (9) there are only two autoregressive (AR) coefficients, $\phi_{12} = 0.525$ and $\phi_{24} = 0.471$ ($\phi_{24} = \phi_{12}\,\phi'_{12}$), with lag values of 12 and 24 respectively. All other AR coefficients are zero. The coefficient y_{t-12} is to account for the seasonal component in y_t, and y_{t-24} is to account for the seasonal component in y_{t-12}. The summation is over the nine causal series ($x_1, x_2, \ldots, x_9$) defined by the pulses. There are no moving average (MA) coefficients, hence $\theta(B) = 0$ on the right-hand side of equations (8) and (9).

When the AR polynomial is multiplied through, the mean parameter is modified to a series trend parameter, and the backorder powers act only on the pulses, i.e., the value of the series at time t is dependent on a linear combination of the value for the pulse, if any, at time t as well as 12 and 24 months previous. Table 2 lists the coefficients, c_i, of the pulse values, x_i. All values are highly significant with p values $\ll 0.001$ and standard errors less than 1.96.

Table 2: USD L501 intervention variables and their statistics

Type	Month	Year	Point	Actual Value	Impact Value, c_i	Standard Error	P Value	T Value
Pulse	Feb	2002	47	5.4037	+3.1241	0.723	.0000	4.32
Pulse	Mar	2002	48	54.8977	+37.5181	0.812	.0000	46.18
Pulse	Mar	2004	72	12.5320	-3.9909	0.863	.0000	-4.62
Pulse	Apr	2005	85	4.5076	+3.9535	0.725	.0000	5.45
Seasonal Pulse	Mar	2006	96	22.5591	+6.7448	1.08	.0000	6.24
Pulse	Nov	2006	104	9.6691	+8.3119	0.790	.0000	10.53
Pulse	Mar	2006	108	29.0319	+5.1069	0.853	.0000	5.98
Pulse	Oct	2007	115	4.6678	+3.1355	0.892	.0006	3.51
Pulse	Feb	2008	119	7.7124	+5.5560	0.887	.0000	6.27

Figure 11 shows how well the model fits the actual data by superimposing the fit (red) on the actual observations (black)[11]. The coefficient of multiple determination, R^2, for the model has a value of 0.986 which implies that 98.6% of the variance in USD L501 expenditures can be explained by equation (9). Since the model is only predictive after lag 24, the first fitted value starts at lag 25. The number of residuals is 96 and the mean squared error (MSE) is 0.879.

[11] Annexes B, C and D display plots of actuals, fitted values and rescaled residuals for USD, GBP and EUR funds respectively.

If it is assumed that the model defined by equation (9) is a true representation of the data, then the rescaled residuals, obtained by dividing the residuals by the estimate of the white noise standard deviation, should resemble a realization of a white noise sequence with variance one.

The rescaled residuals are plotted in Figure 12(a). The mean is -7.08616×10^{-6} and the variance is 1.0. On this basis, there are no indications to doubt the compatibility of the series with unit variance white noise.

Since no more than 5% of the 24 lags fall outside the bounds in the autocorrelation (ACF) plot of the residuals (Figure 12(b)), there is no reason to reject the model on the basis of the autocorrelations.

Finally, Figures 12 (c) and (d) suggest that the assumption of Gaussian white noise is not unreasonable given the linearity of the q-q plot with slight deviation at the tails, and compatibility of the histogram of the residuals with a normal distribution.

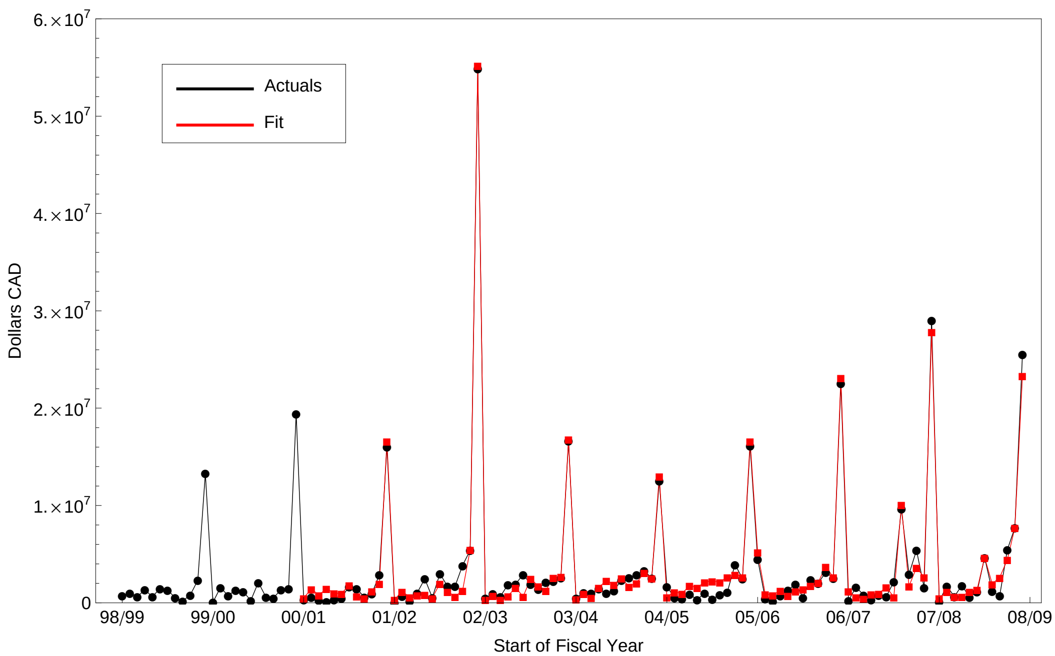

Figure 11: USD L501 fund actual data and model fit

3.3.1 Evaluating the Forecast Ex-Ante

The USD L501 fund model has been evaluated from a statistical point of view by performing various statistical tests on the model and the residuals, but has not been tested for forecasting accuracy. In the evaluation of models by forecast performance, there are a number of dichotomies that need to be examined before a forecasting method can be properly applied [27] . One of the main ones concerns ex-ante versus ex-post evaluation and whether the forecasts can be accurately made before the outcomes have occurred, and evaluated at a later stage when the outcomes are known (ex-ante), or are evaluated against a sub-set of the original dataset retained for in-sample forecasts (ex-post).

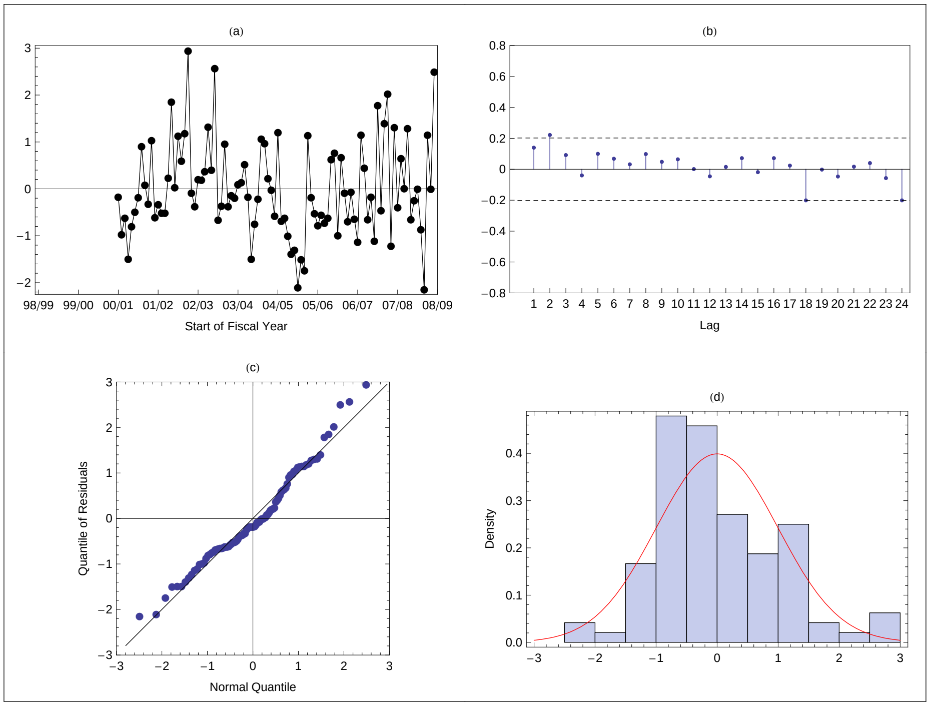

Figure 12: USD L501 rescaled residuals diagnostics

In their paper on evaluating a model through forecast performance, Clements and Hendry [27] conclude that "Out-of-sample [ex-ante] forecast performance is not a reliable indicator of the validity of an empirical model, nor therefore of the economic theory on which the model is based.". Notwithstanding their observation, there is more rationale for using the complete, and defined dataset rather than using a portion other than the realization that a subset would necessarily yield a different model. There is also the reality of the significance of causal variables over the forecast period. In the case of USD L501, a seasonal pulse was detected at point 96 only because 12 months later there was a similar pulse. This would not have been picked up by the ex-post sub-series and consequently the quality of the ex-post forecast would have been underestimated.

Instead, the USD L501 model is completely specified by the 120 data points from April 1998 through March 2008. Given the values for April – July 2008 inclusive, the quality of the forecast is evaluated ex-ante.

Using Filtered Historical Simulation[12] for expenditures, we draw with replacement from the set of past residuals and calculate y_t in equation (9) by substituting ε_t for the sampled value. Running the simulation for 100,000 iteration and accepting only positive values, i.e., $y_t \geq 0$, the results show a distribution of 100,000 results of equation (9). Table 3 displays forecast accuracy statistics relative to the March 2008 (Point 120) origin.

The upper portion of Table 3 displays the immediate comparison of the forecasts (F_t) with

[12]Through bootstrapping a set of residuals. See [1] Section 5.3.

the actuals (A_t). Columns 3 and 4 list the lower and upper 5th percentiles respectively in the distribution of 100,000 sampled expenditures; Column 5 and 6 list the forecast, F_t, taken as the mean of the distribution, and the actual, A_t, as the sum of expenditures for that month; Column 7 lists the percentile value within the distribution where the actual may be found; Column 8 lists the residual (error) as the difference between the actual and the forecast; and, Column 9 lists the percentage, or relative error as the residual divided by the actual multiplied by 100.

There are no observable trends in the percentiles with a reasonable distribution of values on both sides of the median. The percentage error in Point 121 (-1163%) does give some cause for concern without knowing full well why the spending on minor requirements was so low in that month. It may be that the previous month, March 2008, being an end of year aggregation of invoices, left little requirement to start spending so soon in the new fiscal year. In actual fact, the simulation distribution for Point 121 has a sharp peak and is highly skewed-right with a skewness of 1.502 and kurtosis 4.68. Fully 40% of the values show zero spending, so it should not be surprising that the actual falls just left of the median.

Including Point 121, Table 1 shows a positive bias (Cumulative Sum of Forecast Errors, $CFE = 1.3344$) and the forecast has a tendency to under-estimate expenditures. The average error per forecast is $\$7.622 \times 10^5$ CAD, and the sampling distribution of forecast errors has a standard deviation of $\$1.789 \times 10^5$ CAD.

Not including Point 121, the bias is still positive with magnitude $CFE = 1.7886$. The average error per forecast is $\$2.883 \times 10^5$ CAD, or 46.6% of expenditures.

The lower portion of Table 3 largely displays the calculations required to define the tracking signal. A tracking signal allows us to continually monitor the quality of the forecast through time. After each month, a tracking signal value is calculated, and a determination is made as to whether it falls into an acceptable control range. The signal also helps in indicating bias creep by specifying whether the forecast is persistently under or persistently over the actual values. It is computed by dividing the cumulative error by the cumulative mean absolute deviation (*MAD*), i.e.,

$$\text{Tracking Signal (TS)} = \sum (A_t - F_t)/MAD \ . \tag{10}$$

Given control limits of ±2 MADs, Table 3 shows the tracking signal to fall within the bounds of accuracy.

Figure 13 shows how well the forecast (red line) follows the actuals (blue line) within the upper and lower 95th percentile bounds.

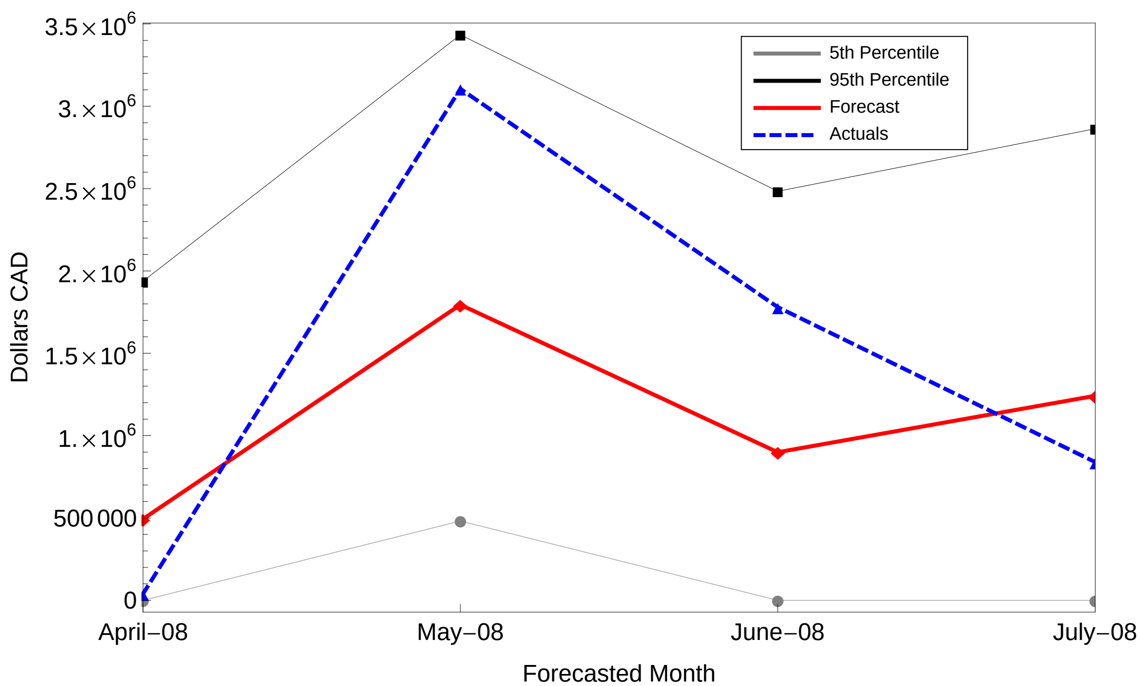

Figure 13: USD L501 comparison of forecast with actuals

Table 3: USD L501 forecast accuracy statistics (dollar values $\times 10^6$)

Point	Date	5^{th} Percentile	95^{th} Percentile	Forecast F_t	Actual A_t	Percentile in Distribution	Residual $A_t - F_t$	Percentage Error, PE_t
121	Apr-08	0.0000	1.9372	0.4933	0.0391	43.7	-0.4542	-1162.93
122	May-08	0.4813	3.4341	1.7950	3.1060	93.7	1.3110	42.21
123	Jun-08	0.0000	2.4840	0.8994	1.7800	82.3	0.8806	49.47
124	Jul-08	0.0000	2.8632	1.2420	0.8390	37.5	-0.4030	-48.03

Point	Date	$\sum(A_t - F_t)$	$\lvert A_t - F_t\rvert$	$\sum\lvert A_t - F_t\rvert$	MAD	Tracking Signal
121	Apr-08	-0.4542	0.4542	0.4542	0.4542	-1.0000
122	May-08	0.8568	1.3110	1.7652	0.8826	0.9707
123	Jun-08	1.7374	0.8806	2.6458	0.8819	1.9699
124	Jul-08	1.3344	0.4030	3.0488	0.7622	1.7506

$$\text{Cumulative Sum of Forecast Errors (CFE)} = \sum(A_t - F_t) = 1.3344$$

$$\text{Mean Absolute Deviation (MAD)} = \frac{1}{4}\sum_{t=1}^{4}|A_t - F_t| = 3.0488/4 = 0.7622$$

$$\text{Mean Squared Error (MSE)} = \frac{1}{4}\sum_{t=1}^{4}(A_t - F_t)^2 = 2.8629/4 = 0.7157$$

$$\text{Standard Deviation of Forecast Errors} = 0.7157/4 = 0.1789$$

$$\text{Mean Absolute Percentage Error (MAPE)} = \frac{1}{4}\sum_{t=1}^{4}|PE_t| = 1302.64/4 = 325.66$$

3.4 The Models

In terms of equations (2) and (9), the general model that defines all funds is given by

$$\sum_{i=1}^{Max\ i} (y_t - \phi_i y_{t-i}) = Constant + \sum_{j=1}^{\#\ interventions} \sum_{i=1}^{Max\ i} \left[c_j \left(1 - \phi_i B^i\right) x_{t,j} \right] + \varepsilon_t \ , \qquad (12)$$

where *Max i* reflects the maximum order of the autoregressive coefficients, ϕ_i; *Constant* is the mean parameter that has been modified to a series trend parameter; and, *# interventions* is the number of interventions that define the model.

3.4.1 The USD Expenditure Models

Tables 4 and 5 define the coefficients and interventions respectively. For example, looking at the Operational Budget (Op Budget) roll-up of the 'L' Funds, we see from Table 4 that the data starts on April 1998 and consequently consists of 120 points through March 2008[13]. There are five AR coefficients with a *Max i* of 25, with values $i = 1$, 12, 13, 24, $25 \neq 0$. All other values are zero. There are no moving average (MA) coefficients.

Table 5 shows there are nine Op Budget interventions, ($j = 1, \ldots, 9$), consisting of seven single pulses, one seasonal pulse and one level shift. Each single pulse occurs at the specified time, t, only. The seasonal pulse occurs at times $t = 108, 120, 132, \ldots$, and the level shift occurs at times $t \geq 92$.

Table 4: USD expenditure models: coefficients

Fund	Begin Month	# Data Points	Constant	ϕ Coefficients$_{(i)}$				
L101	Apr-98	120	0.5370	$0.7210_{(1)}$	$0.9730_{(12)}$	$-0.7020_{(13)}$	—	—
L501	Apr-98	120	0.1020	$0.5250_{(12)}$	$0.4710_{(24)}$	—	—	—
L518	Aug-04	44	0.0697	—	—	—	—	—
C503	Apr-98	120	9.5411	—	—	—	—	—
C113	Apr-98	120	5.9400	$0.3120_{(1)}$	$0.3470_{(12)}$	$-0.1080_{(13)}$	—	—
V511	Feb-07	14	27.3300	—	—	—	—	—
V510	Jul-07	9	0.0619	—	—	—	—	—
C001	Jun-98	118	0.7193	—	—	—	—	—
C107	Aug-00	92	0.0000	$0.2470_{(1)}$	$1.0000_{(12)}$	$-0.2470_{(13)}$	—	—
C160	May-03	59	0.4989	$-0.2530_{(12)}$	—	—	—	—
Op Budget	Apr-98	120	0.5450	$0.4440_{(1)}$	$0.5010_{(12)}$	$-0.2220_{(13)}$	$0.4945_{(24)}$	$-0.2196_{(25)}$
Invest. Cash	Feb-07	14	32.5160	—	—	—	—	—
Other	Jun-98	118	1.7873	—	—	—	—	—

[13] All data ends March 2008 but may start at various periods depending on the size of the sample.

Table 5: USD expenditure models: interventions

Fund	Begin Month	# Data Points	Intervention Coefficients$_{(t)}$[a]								
L101	Apr-98	120	$-7.9482_{(32)}$	$-7.7488_{(36)}$	$20.6060_{(55)}$	$14.2063_{(63)}$	$10.1200_{(77)}$	$8.3263_{(104)}$	$5.4645_{(106)}$	$57.7475_{(119)}$	$31.7138_{(120)}$
L501	Apr-98	120	$3.1241_{(47)}$	$37.5181_{(48)}$	$-3.9909_{(72)}$	$3.9535_{(85)}$	$6.7448_{(96)}$	$8.3119_{(104)}$	$5.1069_{(108)}$	$3.1355_{(115)}$	$5.5560_{(119)}$
L518	Aug-04	44	$0.4600_{(18)}$	$0.6560_{(18)}$	$2.6272_{(19)}$	$1.5896_{(20)}$	$1.0736_{(22)}$	$2.8810_{(28)}$	$1.0478_{(29)}$	$2.7308_{(31)}$	$0.6300_{(36)}$
C503	Apr-98	120	$166.4000_{(36)}$	$13.1744_{(36)}$	$54.2705_{(56)}$	$-82.4595_{(60)}$	$58.9273_{(62)}$	$-124.0200_{(72)}$	$65.5409_{(73)}$	$76.2691_{(81)}$	$-50.824_{(120)}$
C113	Apr-98	120	$14.4536_{(21)}$	$10.4800_{(34)}$	$30.1160_{(36)}$	$89.7983_{(48)}$	$20.1226_{(81)}$	$17.6808_{(85)}$	$13.1765_{(95)}$	$25.6900_{(118)}$	$24.4055_{(120)}$
V511	Feb-07	14	$563.4400_{(6)}$	$199.6100_{(11)}$	—	—	—	—	—	—	—
V510	Jul-07	9	$6.0777_{(1)}$	$49.4219_{(7)}$	$12.6795_{(8)}$	—	—	—	—	—	—
C001	Jun-98	118	$6.3623_{(4)}$	$8.3631_{(7)}$	$10.1855_{(11)}$	$16.6814_{(17)}$	$8.5676_{(41)}$	$7.4942_{(58)}$	$6.8954_{(68)}$	$7.0444_{(69)}$	$20.9209_{(90)}$
C107	Aug-00	92	$-0.5940_{(62)}$	$1.4362_{(64)}$	$-1.0272_{(87)}$	—	—	—	—	—	—
C160	May-03	59	$2.6247_{(23)}$	$2.8772_{(35)}$	$3.0802_{(35)}$	$0.7160_{(46)}$	$1.3384_{(47)}$	$1.7117_{(49)}$	$1.3308_{(52)}$	$1.6814_{(54)}$	$6.0717_{(57)}$
Op Budget	Apr-98	120	$35.0920_{(48)}$	$17.9884_{(55)}$	$16.6461_{(63)}$	$10.4465_{(92)}$	$-11.4907_{(97)}$	$21.0530_{(104)}$	$22.3313_{(108)}$	$64.9911_{(119)}$	$23.9756_{(120)}$
Invest. Cash	Feb-07	14	$564.3900_{(6)}$	$194.7800_{(11)}$	—	—	—	—	—	—	—
Other	Jun-98	118	$7.2950_{(7)}$	$9.1175_{(11)}$	$15.6134_{(17)}$	$9.2006_{(41)}$	$3.3266_{(58)}$	$6.6743_{(68)}$	$4.0348_{(90)}$	$19.1925_{(90)}$	—

[a] Entries in black are single pulses. Entries in red are seasonal pulses. Entries in blue are level shifts.

Therefore, written out in full, the equation that defines the model for USD Operational Budget funds is given by

$$\begin{aligned} y_t & -0.4440\,y_{t-1} - 0.5010\,y_{t-12} + 0.2220\,y_{t-13} - 0.4945\,y_{t-24} + 0.2196\,y_{t-25} \\ = & \quad 0.5450 + [\mathcal{K}] \Big(35.0920\,x_{t=48} + 17.9884\,x_{t=55} + 16.6461\,x_{t=63} \\ + & \quad 10.4465\,x_{t\geq 92} - 11.4907\,x_{t=97} + 21.4907\,x_{t=104} \\ + & \quad 22.3313\,x_{t=108,120,132,\ldots} + 64.9911\,x_{t=119} + 23.9756\,x_{t=120} \Big) \\ + & \quad \varepsilon_t\,, \end{aligned} \tag{13}$$

where the backshift operators in the AR polynomial $[\mathcal{K}] = [1 - 0.4440\,B - 0.5010\,B^{12} + 0.2220\,B^{13} - 0.4945\,B^{24} + 0.2196\,B^{25}]$ act only on the intervention variables, x_t.

Unlike the GBP and EUR expenditures, all USD funds were specified by models, albeit some more defined than others based on available data. The 'V' funds, for example, are not defined well at this stage due to small data samples. They and other funds (even some with large data sets) are defined by ARMA*X* type models where there are no AR or MA components and the exogenous variable, X, are specified by the interventions, x_t [28].

3.4.2 The GBP Expenditure Models

Tables 6 and 7 define the coefficients and interventions respectively for the GBP models. There was insufficient data, at this stage, to define a V510 model. In fact, it may be considered too early to define even a V511 model and consequently the Investment Cash roll-up for GBP.

Table 6: GBP expenditure models: coefficients

Fund	Begin Month	# Data Points	Constant	ϕ Coefficients$_{(i)}$	
L101	Apr-98	120	4.7302	—	—
L501	Apr-98	120	1.3440	$0.1140_{(12)}$	$0.4730_{(24)}$
L518	Aug-04	39	1.9365×10^{-9}	—	—
C503	Apr-98	120	8.1534	—	—
C113	Apr-98	120	14.3910	$0.2690_{(12)}$	—
V511	Feb-07	10	0.4451	—	—
V510	—	—	—	—	—
C001	Jun-98	112	0.0606	—	—
C107	Aug-00	119	2.7600×10^{-3}	$0.3950_{(1)}$	—
C160	May-03	25	0.0431	—	—
Op Budget	Apr-98	120	3.7196	$0.0800_{(12)}$	$0.5620_{(24)}$
Invest. Cash	Feb-07	10	3.3344	—	—
Other	Jun-98	119	0.0699	$0.3760_{(12)}$	—

Table 7: GBP expenditure models: interventions

Fund	Begin Month	# Data Points	Intervention Coefficients$_{(t)}$[a]								
L101	Apr-98	120	$-4.3233_{(1)}$	$2.0164_{(19)}$	$-3.4610_{(21)}$	$5.2213_{(24)}$	$10.5262_{(54)}$	$19.4666_{(63)}$	$4.9236_{(67)}$	$13.9481_{(107)}$	$28.6726_{(120)}$
L501	Apr-98	120	$9.7831_{(27)}$	$8.7606_{(29)}$	$11.3788_{(47)}$	$12.6791_{(48)}$	$11.7481_{(65)}$	$8.7247_{(69)}$	$11.3750_{(72)}$	$17.8642_{(96)}$	$-3.8527_{(108)}$
L518	Aug-04	39	$0.3290_{(1)}$	$0.1100_{(28)}$	$0.1090_{(29)}$	$2.5743_{(39)}$	—	—	—	—	—
C503	Apr-98	120	$54.7392_{(53)}$	$25.3673_{(60)}$	$61.5471_{(84)}$	$33.3894_{(90)}$	$29.6388_{(94)}$	$64.2522_{(96)}$	$35.5193_{(102)}$	$34.8315_{(107)}$	$51.6846_{(111)}$
C113	Apr-98	120	$107.9600_{(36)}$	$-103.9800_{(48)}$	$196.1400_{(60)}$	$166.6200_{(61)}$	$61.2412_{(79)}$	$100.3700_{(80)}$	$74.5293_{(81)}$	$-49.5667_{(96)}$	$-102.6300_{(120)}$
V511	Feb-07	10	$5.8471_{(2)}$	$2.0096_{(4)}$	$4.2483_{(5)}$	$13.5051_{(6)}$	—	—	—	—	—
V510	—	—	—	—	—	—	—	—	—	—	—
C001	Jun-98	112	$0.3170_{(27)}$	$0.9610_{(32)}$	$18.2028_{(39)}$	$0.3080_{(43)}$	$0.5240_{(47)}$	$0.5230_{(68)}$	$8.3558_{(90)}$	$8.2901_{(96)}$	$12.2348_{(99)}$
C107	Aug-00	119	$0.1630_{(3)}$	$0.2760_{(4)}$	$0.0688_{(5)}$	$0.0692_{(7)}$	$0.0568_{(9)}$	$0.0333_{(10)}$	$0.0862_{(11)}$	$0.0768_{(18)}$	$0.0219_{(76)}$
C160	May-03	25	$1.9688_{(1)}$	$0.4470_{(4)}$	$0.1330_{(5)}$	$0.0929_{(11)}$	$0.0879_{(19)}$	$0.1610_{(23)}$	$0.0833_{(25)}$	—	—
Op Budget	Apr-98	120	$12.6578_{(27)}$	$23.9193_{(48)}$	$12.3937_{(54)}$	$19.0379_{(63)}$	$-12.5830_{(76)}$	$12.0438_{(92)}$	$13.7277_{(107)}$	$-13.9745_{(108}$	—
Invest. Cash	Feb-07	10	$-3.3344_{(3)}$	$10.6158_{(6)}$	$-3.3344_{(7)}$	$8.7150_{(10)}$	—	—	—	—	—
Other	Jun-98	119	$0.7990_{(39)}$	$18.1758_{(46)}$	$0.3400_{(50)}$	$0.4950_{(83)}$	$1.9548_{(95)}$	$8.3564_{(97)}$	$8.1302_{(103)}$	$12.6219_{(106)}$	—

[a] Entries in black are single pulses. Entries in red are seasonal pulses. Entries in blue are level shifts.

3.4.3 The EUR Expenditure Models

Tables 8 and 9 define the coefficients and interventions respectively for the EUR models. As for the GBP expenditures, there was insufficient data, at this stage, to define a V511 model. Furthermore, there were only eight data points to define V510 and consequently the Investment Cash roll-up.

Table 8: EUR expenditure models: coefficients

Fund	Begin Month	# Data Points	Constant	ϕ Coefficients$_{(i)}$	
L101	Dec-99	100	0.3214	$0.3550_{(1)}$	$0.3430_{(2)}$
L501	Jul-00	93	0.0612	—	—
L518	Dec-06	16	3.1500×10^{-3}	$0.8550_{(1)}$	—
C503	Sep-01	79	0.6811	—	—
C113	Jun-00	94	0.9320	$0.3510_{(12)}$	—
V511	—	—	—	—	—
V510	Aug-07	8	2.9243	—	—
C001	Oct-00	90	2.6210	$0.5500_{(12)}$	—
C107	Nov-01	77	0.0124	—	—
C160	Oct-03	54	1.6160×10^{-4}	$0.6400_{(1)}$	—
Op Budget	Dec-99	100	0.4976	$0.4000_{(1)}$	$0.1920_{(3)}$
Invest. Cash	Aug-07	8	2.6310	$0.5510_{(12)}$	—
Other	Oct-00	90	2.3741	—	—

Table 9: EUR expenditure models: interventions

Fund	Begin Month	# Data Points	Intervention Coefficients$_{(t)}$[a]								
L101	Dec-99	100	$1.9254_{(28)}$	$2.5654_{(63)}$	$1.4909_{(86)}$	$14.1163_{(90)}$	$3.4778_{(91)}$	$13.2841_{(97)}$	$23.6475_{(100)}$	—	—
L501	Jul-00	93	$0.9650_{(16)}$	$2.1575_{(21)}$	$1.0507_{(30)}$	$0.9910_{(31)}$	$0.6140_{(33)}$	$1.1432_{(33)}$	$0.4480_{(56)}$	$0.8320_{(60)}$	$0.4650_{(77)}$
L518	Dec-06	16	$0.6510_{(3)}$	$0.2860_{(4)}$	$0.0350_{(10)}$	$0.0755_{(13)}$	—	—	—	—	—
C503	Sep-01	79	$16.8137_{(16)}$	$4.5609_{(33)}$	$11.8565_{(40)}$	$30.1179_{(43)}$	$9.9802_{(61)}$	$8.2113_{(62)}$	$4.3876_{(75)}$	$3.1367_{(77)}$	$4.9940_{(79)}$
C113	Jun-00	94	$2.5877_{(20)}$	$21.5384_{(22)}$	$8.3216_{(34)}$	$4.1417_{(44)}$	$-3.5062_{(46)}$	$2.6384_{(58)}$	$-4.6389_{(70)}$	$4.8915_{(90)}$	$4.1808_{(92)}$
V511	—	—	—	—	—	—	—	—	—	—	—
V510	Aug-07	8	$5.3836_{(1)}$	$-2.4309_{(6)}$	—	—	—	—	—	—	—
C001	Oct-00	90	$13.4533_{(18)}$	$11.7765_{(39)}$	$26.4958_{(42)}$	$14.3250_{(43)}$	$14.7666_{(53)}$	$14.7931_{(74)}$	$39.3879_{(77)}$	$13.7466_{(87)}$	$28.4118_{(90)}$
C107	Nov-01	77	$0.0282_{(10)}$	$0.0828_{(16)}$	$0.0757_{(21)}$	$0.0420_{(25)}$	$0.0364_{(27)}$	$-0.0572_{(33)}$	$0.0521_{(40)}$	$0.1350_{(48)}$	$0.0493_{(51)}$
C160	Oct-03	54	$0.0038_{(3)}$	$0.0036_{(4)}$	$0.0099_{(32)}$	$0.2250_{(41)}$	$0.2290_{(52)}$	—	—	—	—
Op Budget	Dec-99	100	$3.1028_{(28)}$	$3.6478_{(63)}$	$13.1950_{(90)}$	$13.7308_{(97)}$	$23.1701_{(100)}$	—	—	—	—
Invest. Cash	Aug-07	8	$5.9338_{(1)}$	$73.4408_{(8)}$	—	—	—	—	—	—	—
Other	Oct-00	90	$13.4347_{(18)}$	$11.7824_{(39)}$	$26.4775_{(42)}$	$14.3261_{(43)}$	$14.8001_{(53)}$	$14.7894_{(74)}$	$39.6037_{(77)}$	$13.7320_{(87)}$	$28.4078_{(90)}$

[a] Entries in black are single pulses. Entries in red are seasonal pulses.

4 The Currency Models

This section describes the models for forecasting the foreign exchange rates for the USD, GBP and EUR currencies. For a complete background on the mechanisms to specify and validate currency models, the reader is referred to section 4 of [1]. To follow the logical progression of model development, the key points from [1] will be restated.

4.1 The Returns

Financial returns are known to exhibit certain stylized properties that are common across a wide range of markets and time periods. Examples of these properties are volatility clustering, the leptokurtic[14] distribution of returns, high autocorrelation of squared returns and no autocorrelation of raw returns [29, 30].

Extreme values are found in the tails of the distribution where "fat tails" can be used to explain the dynamics of large price fluctuations that are much higher then predictable by the normal distribution [31]. In such cases, distributions such as the Generalized Error or Student's *t* can be used, where, in the case of the latter, the degrees of freedom parameter, along with the rest of the model parameters, can be estimated using maximum likelihood. The degrees of freedom estimate will control the fatness of the tails fitted from the model.

Figure 14 shows the time series plots of the daily closing rates (a–c) and continuously compounded returns (d–f) of the three currencies. The logarithm of the exchange rates are generally considered to follow a random walk model and as such, the rates are not mean-reverting[15] [32].

The time series of returns in Figure 14 (d-f) show clear evidence of volatility clustering. Periods of high volatility, e.g., beginning FY 03/04 in Figure 14 (d), are clustered and distinct from periods of low volatility, e.g., during FY 96/97. Measuring volatility in terms of variance, the time series of currency returns implies that variance, σ_t^2, changes with time or is heteroscedastic.

Return statistics are given in Table 10 for both returns and squared returns. The mean of each return series is effectively zero. The skewness, a measure of lack of symmetry, shows CAD/GBP slightly skewed left and CAD/USD and CAD/EUR skewed right with CAD/EUR more so than CAD/USD. The excess kurtosis relative to normal shows reasonable peaking for all three currencies as a consequence of leptokurtic distributions, with CAD/USD showing the highest peak around the mean. All three currencies show no autocorrelation evidenced by a low Ljung-Box statistic.

Squared returns, on the other hand, do show a strong autocorrelation (high Ljung-Box, low p-value) as the null hypothesis fails indicating the data is not independent. Autocorrelation in the squared returns implies autocorrelation in variances.

[14] The condition for a probability density curve to have fatter tails and a higher peak at the mean than the normal distribution.

[15] Mean reversion is the tendency for a stochastic process to remain near, or tend to return over time to a long-run average value.

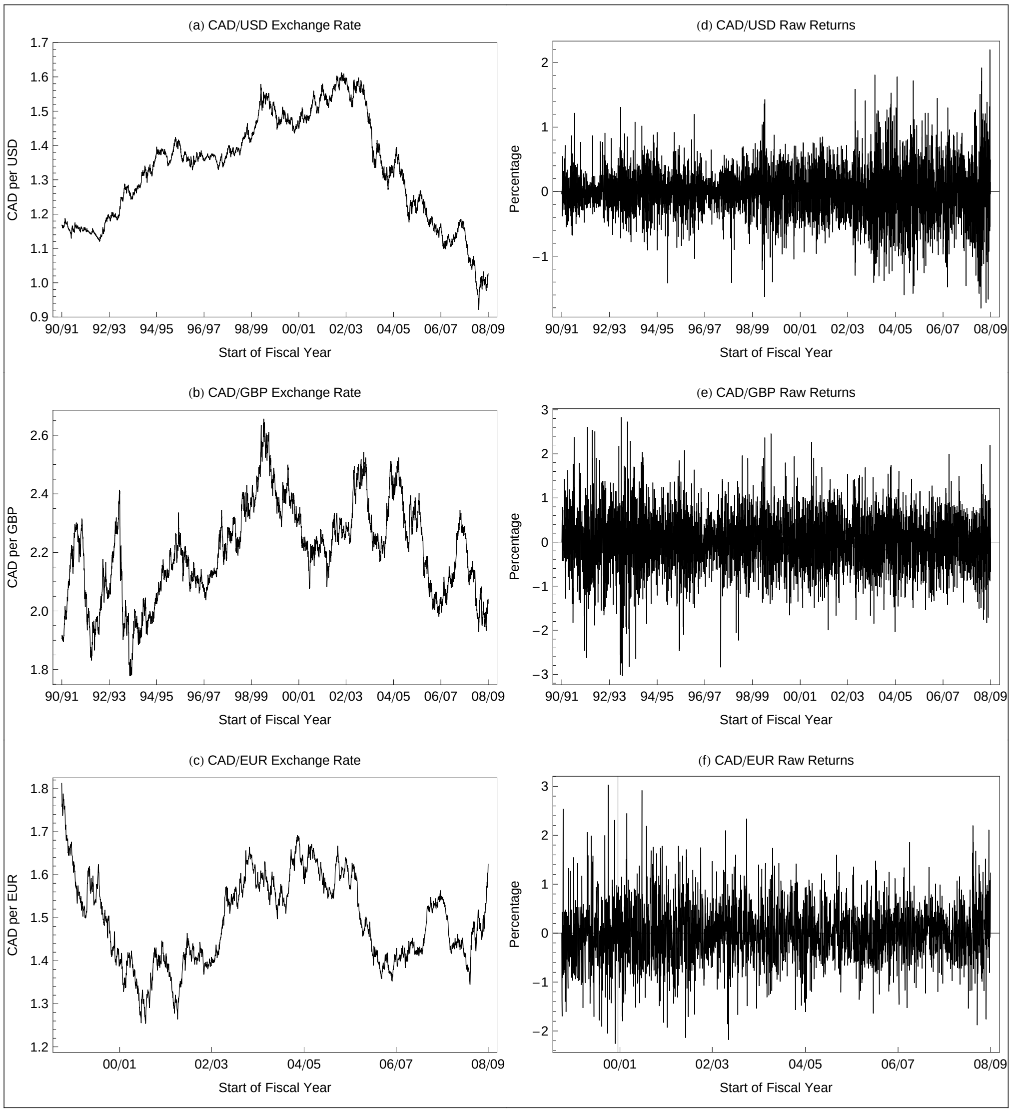

Figure 14: (a–c): Time plots of CAD/USD, GBP and EUR exchange rates and (d–f): raw returns. Based on 18 years, or 4515 daily observations for CAD/USD and CAD/GBP; and 9.25 years, or 2320 daily observations for CAD/EUR.

Table 10: Return and squared return statistics

	CAD/USD	CAD/GBP	CAD/EUR
Mean	-2.86×10^{-5}	1.52×10^{-5}	-4.72×10^{-5}
Skewness	0.0134	-0.0913	0.2422
Excess kurtosis	2.2456	1.6031	1.1795
Ljung-Box(20) (p-value)	26.497 (0.1500)	29.872 (0.0720)	17.318 (0.6322)
Ljung-Box^2(20) (p-value)	1849.1 (0.0000)	551.2 (0.0000)	96.265 (0.0000)

4.2 The GARCH(1,1) Variance Models

There are two aspects to the problem of calculating a VaR and determining the foreign exchange risk to the department; first, we need to model the expenditures for each fund (Section 3), and secondly, we need to develop models for the financial returns series that accurately model the characteristics of each currency such as time-varying volatilities, volatility clustering and non-normal distributions.

GARCH, *Generalized Autoregressive Conditional Heteroskedasticity*[16], models have become important in the analysis of time series ever since Bollerslev introduced them in 1986 [33] as a generalization of Engle's ARCH (*Autoregressive Conditional Heteroskedasticity*) model [34]. Since then, the family of GARCH-type models has grown at a phenomenal rate.

The standard GARCH(p,q) model, where the conditional variance, σ_t, is parameterized to depend upon q lags of the squared return and p lags of the conditional variance is defined by

$$\sigma_t^2 = \omega + \sum_{i=1}^{q} \alpha_i r_{t-i}^2 + \sum_{j=1}^{p} \beta_j \sigma_{t-j}^2 \; , \tag{14}$$

where we assume non-normality of the returns distribution and let

$$r_t = \sigma_t z_t \quad \text{with } z_t \sim \tilde{t}(d) \; , \tag{15}$$

where z_t is the error term now defined by the standardized $t(d)$ distribution, and the conditional distribution of r_t coincides with the distribution of z_t.

If $p = q = 1$, the model becomes the basic GARCH$(1,1)$ model which has been extensively used to model the main statistical characteristics of a wide range of assets, i.e.,

$$\boxed{\sigma_t^2 = \omega + \alpha\, r_{t-1}^2 + \beta\, \sigma_{t-1}^2 \; .} \tag{16}$$

In equation (16), the parameters ω, α, and β are unknown constants that satisfy $\omega > 0$, $\alpha \geq 0$, and $\beta \geq 0$ to ensure positivity of the conditional variance, and $\alpha + \beta < 1$ is a necessary and sufficient condition to ensure covariance stationary.

4.2.1 Maximum Likelihood Estimation (MLE) with $\tilde{t}(d)$

Let $\{r_1, \ldots, r_T\}$ be a series of T observations generated by a GARCH(1,1) process given by equation (15). The goal here is to estimate directly the distribution of r_{T+k} and σ_{T+k} conditional on the available data. The unknown parameters in the GARCH(1,1) process are normally estimated by quasi-maximum likelihood maximizing the normal log-likelihood function. However, since the assumption of normality is violated, albeit moderately, in the distribution of returns, we instead choose to maximize the log-likelihood function of the $\tilde{t}(d)$ distribution.

[16] Autoregressive describes a feedback mechanism that incorporates past observations into the present; Conditional implies a dependence on observation of the immediate past; and, Heteroskedastic refers to time-varying variance or volatility.

Following Christoffersen [35], the $\tilde{t}(d)$ density is defined by

$$f_{\tilde{t}(d)}(z;d) = \frac{\Gamma((d+1)/2)}{\Gamma(d/2)\sqrt{\pi(d-2)}}(1+z^2/(d-2))^{-(1+d)/2} , \tag{17}$$

where d are the degrees of freedom and must be greater than 2 for the distribution to be well defined; z is the random variable with mean zero and standard deviation one; and, $\Gamma(*)$ is the standard gamma function.

If we consider the standardized return as a random variable defined by equation (15), i.e., $z_t = r_t/\sigma_t$, then the log-likelihood of the sample of returns is given by

$$\begin{aligned} \ln L &= \sum_{t=1}^{T} \ln(f(r_t;d)) - \sum_{t=1}^{T} \ln(\sigma_t^2)/2 \\ &= T\{\ln(\Gamma((d+1/2)) - \ln(\Gamma(d/2)) - \ln(\pi)/2 - \ln(d-2)/2\} \\ &\quad -\frac{1}{2}\sum_{t=1}^{T}(1+d)\ln(1+(r_t/\sigma_t)^2/(d-2)) - \sum_{t=1}^{T}\ln(\sigma_t^2)/2 , \end{aligned} \tag{18}$$

where the last term in equation (18) takes into account the variance, and the unknown parameters (ω, α, β, d) are estimated through maximizing equation (18). Once the values of (ω, α, β) are estimated by MLE, the conditional variances are estimated by equation (16).

4.2.2 Validation of Non-Normality Assumption

Given that we have modelled the GARCH(1,1) process by assuming that the $\tilde{t}(d)$ distribution best models the non-normality of the returns, we need to validate our assumption through comparison of the return quantiles. This is best conducted by plotting the return quantiles against normal and $\tilde{t}(d)$ quantiles on quantile-quantile (QQ) plots.

The quantile-quantile, QQ plot, is a graphical technique for determining if two data sets are defined by a common distribution. For example, if the returns were defined by a normal distribution, plotting the quantiles of the standardized returns against the quantiles of the normal distribution should define a line on a 45-degree angle. Any deviations from the 45-degree line indicate that the returns are not well described by the assumed distribution, be it normal or $\tilde{t}(d)$.

Figure 15 plots the quantiles of the three currency returns standardized by the unconditional standard deviation against the normal distribution (a-c); standardized by the GARCH(1,1) against the normal distribution (d-f); and, standardized by the GARCH(1,1)$-\tilde{t}(d)$ against the Student's t distribution[17].

Comparing the CAD/USD panel, Figure 15 (a, d, g), we note that both the left and the right tails are best fit with the $\tilde{t}(d)$ distribution.

[17] The quantile of the standardized $\tilde{t}(d)$ distribution is not easily found. Consequently, the conventional Student's t(d) was substituted.

For CAD/GBP, Figure 15 (b, e, h) we note that the left tail is best fit with the $\tilde{t}(d)$ distribution but the right tail is best fit with the normal distribution. Since we are mainly interested in forecasting a loss, it is more important to focus on the left tail and consequently standardizing the returns with the GARCH model whose coefficients are derived through maximizing the log-likelihood of the $\tilde{t}(d)$ distribution.

For CAD/EUR, the model fits the right tail better with $\tilde{t}(d)$ Figure (15 (f, i)) but at the cost of the left tail, which exhibits significant deviation from the 45-degree line. Therefore, the results indicate that the left tail is best fit by the normal distribution and not $\tilde{t}(d)$. That being said, it is entirely possible that the data used may simply not have enough extreme observations in the sample (and generate fat-tails) even though they could exist. The Euro is a relatively new currency and most likely a much larger sample size would provide justification for fitting this model, in particular the left tail, with $\tilde{t}(d)$. In the interim, the CAD/EUR GARCH model is specified by maximizing the standard maximum log-likelihood

$$\ln L = \sum_{t=1}^{T} [-\frac{1}{2}\ln(2\pi) - \frac{1}{2}\ln(\sigma_t^2) - \frac{1}{2}\frac{r_t^2}{\sigma_t^2}] , \qquad (19)$$

where r_t is defined by equation (15) with the error distribution now independently and identically normally distributed with mean equal to zero and variance equal to one, i.e.,

$$z_t \sim i.i.d.\ N(0,1) .$$

Table 11 provides the GARCH coefficients and degrees of freedom, d, for the $\tilde{t}(d)$ distribution. The parameters were estimated on 4515 daily observations between 01 April 1990 and 31 March 2008 for CAD/USD and CAD/GBP; and, 2320 daily observations between 04 January 1999 and 31 March 2008 for CAD/EUR. Both CAD/USD and CAD/GBP currency returns are best fit with a GARCH(1,1) whose parameters are estimated from the standardized t-distribution. For CAD/EUR, the currency returns are best fit with a GARCH(1,1) whose parameters are estimated from a normal distribution; although it is acknowledged that close scrutiny of extreme observations is required to ensure optimal model specification.

In Table 11, the sum $\alpha+\beta$, also known as the *persistence* of the model, determines the rate of reversion of the model to its long-run mean variance. A high persistence, $\alpha+\beta$ close to one, implies that shocks to the conditional variance *persist* for a long time affecting future forecasts of volatility, but eventually the long-run forecast will revert back to the long-run average variance.

Table 11: Coefficients for the GARCH(1,1) models

Return	ω	α	β	d	$\alpha+\beta$
CAD/USD	1.6535×10^{-8}	0.04112	0.9589	9.2695	0.9999
CAD/GBP	2.0091×10^{-7}	0.03596	0.9959	8.6992	0.9959
CAD/EUR	7.1720×10^{-8}	0.017624	0.9984	—	0.9984

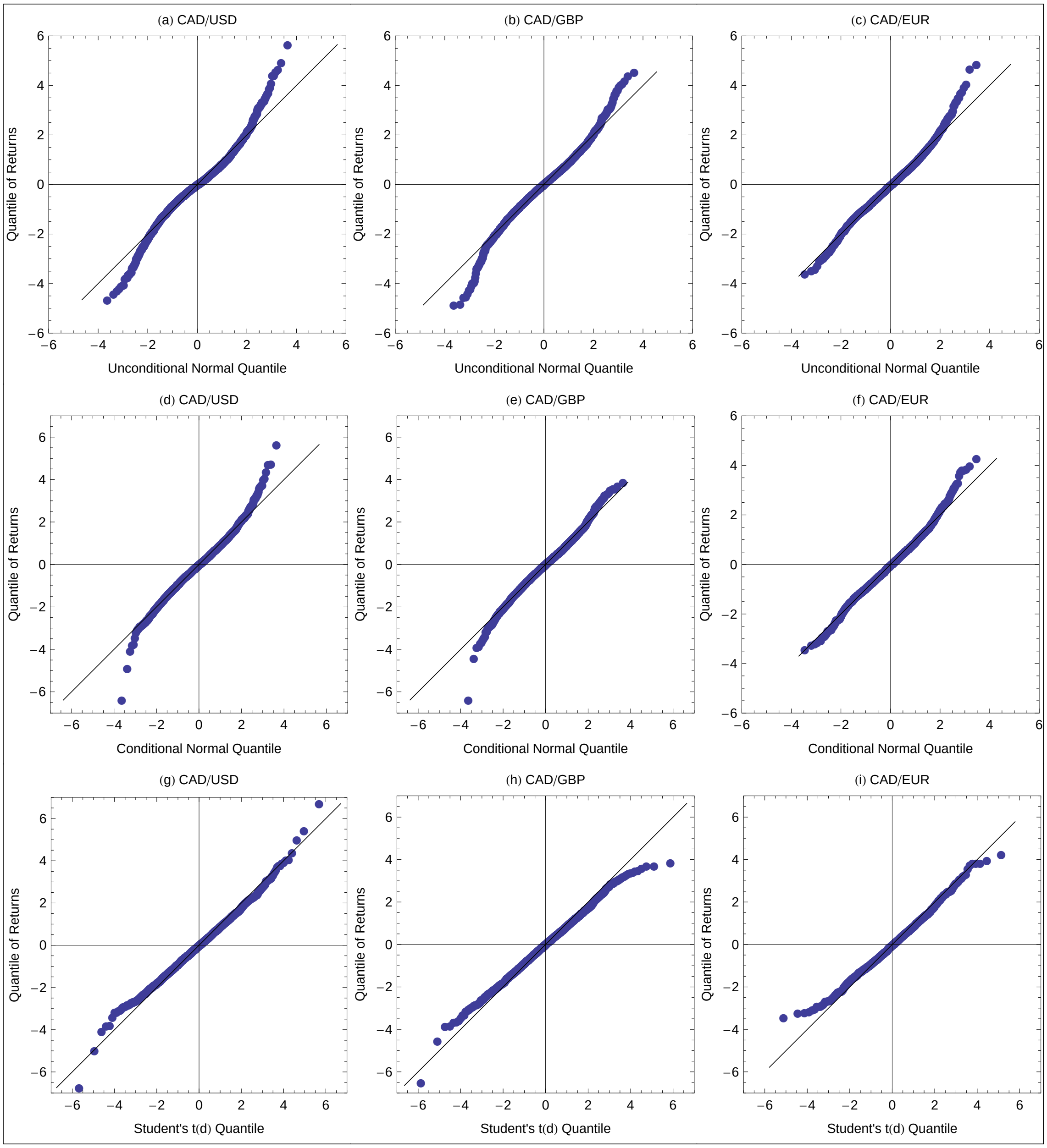

Figure 15: Quantile-Quantile plots of daily CAD/USD, CAD/GBP and CAD/EUR returns (a-c); (d-f) returns standardized by GARCH(1,1) against the normal distribution; (g-i) returns standardized by GARCH(1,1) against the student-t distribution

5 The Departmental VaR Model

The overall aim of this study is to develop a model for which departmental financial analysts could use to forecast loss or gains on exchange and the implications on local budgets that have to, a priori, apportion funding for future contract invoices. Furthermore, these forecasts should be limited to no more than three months (one quarter) since volatility is effectively not forecastable beyond a certain period[18]. In the previous sections, models were built and validated for forecasting 10 major departmental funds and their aggregates as well as the conditional variances for the three currencies of interest. In this section, all the models are assembled to build a VaR model for the department that allows a user to forecast the maximum expected loss from adverse exchange rate fluctuations over the budget year.

5.1 Filtered Historical Simulation For Returns

In [1], it was determined that Filtered Historical Simulation (FHS) was the preferred method for representing actual market behaviour as it captures all possible values of the historical distribution of price returns, in particular the tail events critical to VaR calculations, with the least number of assumptions about the statistical properties of future price changes.

Filtered Historical Simulation (FHS) is non-parametric in the sense that the simulation imposes no structure on the distribution of returns [37, 14]. There is no need to make any distributional assumptions, whether normal or $\tilde{t}(d)$, on the standardized returns of the currency exchanges.

Following [35], we start the process by considering the set of past returns $\{r_{t+1-\tau} : \tau = 1,2,\ldots,T\}$ where $T = 4514$ and 2319 for CAD/(USD, GBP) and CAD/EUR respectively. From equation (15), we can write the one-day ahead return as the product of the estimated standard deviation and the error term, i.e.,

$$r_{t+1} = \sigma_{t+1} z_{t+1} \,, \tag{20}$$

where σ_{t+1} is defined through the GARCH variance equation (16), already calibrated using eighteen years of historical data, to be

$$\sigma_{t+1} = \left[\omega + \alpha r_t^2 + \beta \sigma_t^2\right]^{1/2} \,, \tag{21}$$

with parameters (ω, α, β) defined in Table 11. Using the data set $\{r_{t+1-\tau} : \tau = 1,2,\ldots,T\}$ we can now estimate the model parameters and calculate the set of realized standardized returns, $\{\hat{z}_{t+1-\tau} : \tau = 1,2,\ldots,T\}$, defined by

$$\hat{z}_{t+1-\tau} = r_{t+1-\tau}/\sigma_{t+1-\tau} \,, \qquad \text{for } \tau = 1,2,\ldots,T \tag{22}$$

Therefore, given actual returns up to time t (31 March 2008), we can immediately evaluate the GARCH variance and equation (21) for time $t+1$. To compute hypothetical returns for

[18] As stated in [1]:"[The forecast] is not very accurate if the horizon of interest is more than 20 days, since volatility is effectively not forecastable beyond that limit [36]. Therefore, forecasts up to one quarter should be treated with varying degrees of confidence.

tomorrow, 01 April 2008, we draw with replacement from the set of past standardized residuals, $\{\hat{z}_{t+1-\tau} : \tau = 1, 2, \ldots, T\}$, through sampling a discrete uniform distribution of elements consisting of the $\tau = 1, 2, \ldots, T$ standardized returns defined by equation (22). The estimated exchange rate, P_{t+1}, on 01 April 2008 is then defined to be

$$P_{t+1} = e^{\,r_{t+1}} P_t \ , \tag{23}$$

where P_t is defined as the exchange rate on day t.

To illustrate the process for the next 264 trading days (12 months @ 22 trading days per month) ending 31 March 2009, consider the algorithm described in Figure 16. The return and conditional variance on the last day of actual data (31 March 2006) starts the simulation. After each 22-day trading period, the estimated exchange rate at that time is captured for each iteration and used in a subsequent calculation for the VaR based on equation (1). As depicted in Figure 17, days 22, 44, etc., correspond to 30 April 2008, 31 May 2008, etc., respectively. Therefore, the end result is 10,000 sequences of hypothetical daily returns for day $t+1$ through day $t+264$.

Figure 16: The FHS process for returns

Iterations $\downarrow$ **Days** $\longrightarrow$

$$(r_{31/03}, \sigma^2_{31/03}) \Rightarrow \left\{ \begin{array}{ccc} (\hat{z}_{1,1}, \sigma^2_{1,1}) \rightarrow r_{1,1} \rightarrow P_{1,1} & \cdots & (\hat{z}_{1,264}, \sigma^2_{1,264}) \rightarrow r_{1,264} \rightarrow P_{1,264} \\ (\hat{z}_{2,1}, \sigma^2_{2,1}) \rightarrow r_{2,1} \rightarrow P_{2,1} & \cdots & (\hat{z}_{2,264}, \sigma^2_{2,264}) \rightarrow r_{2,264} \rightarrow P_{2,264} \\ \vdots & \vdots & \vdots \\ \vdots & \vdots & \vdots \\ (\hat{z}_{10k,1}, \sigma^2_{10k,1}) \rightarrow r_{10k,1} \rightarrow P_{10k,1} & \cdots & (\hat{z}_{10k,264}, \sigma^2_{10k,264}) \rightarrow r_{10k,264} \rightarrow P_{10k,264} \end{array} \right.$$

Figure 17: Extraction of monthly exchange rates

$$\left| \begin{array}{ccccccccc} P_{1,1} & P_{1,2} & \cdots & P_{1,22} & \cdots & P_{1,44} & \cdots & P_{1,264} \\ P_{2,1} & P_{2,2} & \cdots & P_{2,22} & \cdots & P_{2,44} & \cdots & P_{2,264} \\ \vdots & \vdots & \vdots & \vdots & & & & \\ \vdots & \vdots & \vdots & \vdots & & & & \\ P_{10k,1} & P_{10k,2} & \cdots & P_{10k,22} & \cdots & P_{10k,44} & \cdots & P_{10k,264} \end{array} \right|$$

$$\begin{array}{ccccc} \downarrow & & \downarrow & \cdots & \downarrow \\ \text{VaR} & & \text{VaR} & & \text{VaR} \end{array}$$

5.1.1 The Excel Model for Returns

The above section is prototype modelled in Excel and a sample of the main GARCH worksheet is shown in Figure 18. While the actual historical data goes from row 4 to row 4518 (4515 daily rate values for CAD/USD and CAD/GBP), the sample shown cuts off at row 9 and continues at row 4507. The dashed red line at row 4518 signifies the division between actual and forecasted values. Therefore, rows 4519 through 4540 show the sampled[19] forecasted results for each of 22 trading days in April 2008, with the exchange rate on the 22nd trading day (highlighted in yellow) extracted for the VaR calculation as shown in Figure 17.

There are 14 columns in Figure 18 labelled A through N. From row 4 through 4518:

- Column A displays the market trading date (weekends and holidays are not included) from 02 April 1990 through 31 March 2008.
- Columns B through D display the historical daily exchange rates, P_t (CAD/EUR rates don't start until row 2199 - 4th January 1999).
- Columns E through F display the currency returns, r_t, defined by $r_t = \ln P_t - lnP_{t-1}$.
- Columns H through J display the standardized returns, standardized by the GARCH variance, σ_t^2, i.e., $z_t = r_t/\sigma_t$.
- Columns K through N display the calculations applicable to the CAD/USD columns only (calculations for CAD/GBP and CAD/EUR are actually displayed from Column O).
  - Column K displays the Conditional (GARCH) Variance calculation (equation (21)), where the starting value on 3rd April 1990 is given by the unconditional variance of the return series, i.e., in Excel: $VAR(E5 : E4518)$.
  - Column L displays the $\tilde{t}(d)$ maximum likelihood estimation calculation of equation (18), where the sum of the log-likelihood function (MLE) is displayed in cell (row 9, column N) and the degrees of freedom parameter one row above.
  - Column N also displays the GARCH parameters (ω, α, β) that need to be adjusted together with d such that the MLE is maximized conditional on the persistence, $\alpha + \beta$ being less than one.

The forecasting portion of Figure 18 (from row 4519) simply displays all calculations starting with the evaluation of "… the GARCH variance and equation (21) for time $t+1$." The hypothetical returns are calculated through equation (20) by first drawing with replacement from the set of past standardized residuals, $H5 : H4518$, through sampling a discrete uniform distribution of elements. The forecasted exchange rate is then calculated through equation (23).

[19]This would be one of 10,000 samples as depicted in Figure 16.

	A	B	C	D	E	F	G	H	I	J	K	L	M	N
1	**Estimating the GARCH(1,1) model for the variance of each currency**													
2		**Daily Rates**			**Return**			**GARCH Standardized Return**			**CAD/USD**			
3	**Date**	**CAD/USD**	**CAD/GBP**	**CAD/EUR**	**CAD/USD**	**CAD/GBP**	**CAD/EUR**	**CAD/USD**	**CAD/GBP**	**CAD/EUR**	**Conditional Variance**	**Likelihood**	**GARCH Final Values**	
4	02-Apr-90	1.1678	1.9056										**ω**	3.21E-08
5	03-Apr-90	1.1673	1.9072		-0.04%	0.08%		-0.1099	0.1346		1.52E-05	4.7300	**α**	5.13E-02
6	04-Apr-90	1.1678	1.9167		0.04%	0.50%		0.1127	0.8114		1.44E-05	4.7544	**β**	9.49E-01
7	05-Apr-90	1.1687	1.9178		0.08%	0.06%		0.2078	0.0942		1.37E-05	4.7567	**α+β**	0.999999
8	06-Apr-90	1.1643	1.9098		-0.38%	-0.42%		-1.0420	-0.6990		1.31E-05	4.0687	**d**	8.14578954
9	09-Apr-90	1.1616	1.9049		-0.23%	-0.26%		-0.6392	-0.4334		1.32E-05	4.5149	**MLE**	19193.6657
4507	13-Mar-08	0.9856	1.999	1.5336	-0.47%	-0.06%	0.01%	-0.6873	-0.0795	0.0189	4.59E-05	3.8472		
4508	14-Mar-08	0.9862	2.0028	1.5402	0.06%	0.19%	0.43%	0.0910	0.2798	0.6368	4.47E-05	4.1930		
4509	17-Mar-08	0.9993	1.997	1.573	1.32%	-0.29%	2.11%	2.0254	-0.4350	3.1662	4.24E-05	1.8867		
4510	18-Mar-08	0.9932	2.0091	1.5686	-0.61%	0.60%	-0.28%	-0.8726	0.9203	-0.3626	4.92E-05	3.6166		
4511	19-Mar-08	1.0153	1.9913	1.5685	2.20%	-0.89%	-0.01%	3.1548	-1.3599	-0.0084	4.87E-05	-0.2468		
4512	20-Mar-08	1.0234	2.0356	1.583	0.79%	2.20%	0.92%	0.9428	3.3119	1.2448	7.10E-05	3.3497		
4513	24-Mar-08	1.0179	2.0254	1.5703	-0.54%	-0.50%	-0.81%	-0.6410	-0.6469	-1.0814	7.07E-05	3.6741		
4514	25-Mar-08	1.0173	2.0351	1.5871	-0.06%	0.48%	1.06%	-0.0712	0.6232	1.4283	6.86E-05	3.9814		
4515	26-Mar-08	1.0188	2.0377	1.6044	0.15%	0.13%	1.08%	0.1826	0.1687	1.4309	6.51E-05	3.9864		
4516	27-Mar-08	1.0184	2.039	1.603	-0.04%	0.06%	-0.09%	-0.0499	0.0860	-0.1133	6.19E-05	4.0344		
4517	28-Mar-08	1.0215	2.0252	1.6046	0.30%	-0.68%	0.10%	0.3965	-0.9341	0.1325	5.88E-05	3.9468		
4518	31-Mar-08	1.0265	2.0407	1.6244	0.49%	0.76%	1.23%	0.6510	1.0526	1.6667	5.63E-05	3.7792		
4519		1.0301	2.0380	1.6230	0.35%	-0.13%	-0.09%	0.4716	-0.1829	-0.1166	5.46E-05			
4520		1.0323	2.0282	1.6098	0.22%	-0.48%	-0.81%	0.3009	-0.6745	-1.0961	5.25E-05			
4521		1.0417	2.0229	1.6302	0.91%	-0.26%	1.26%	1.2794	-0.3729	1.6938	5.01E-05			
4522		1.0471	2.0058	1.6494	0.51%	-0.85%	1.17%	0.7146	-1.2319	1.5267	5.17E-05			
4523		1.0496	2.0096	1.6404	0.24%	0.19%	-0.55%	0.3349	0.2705	-0.6993	5.05E-05			
4524		1.0467	2.0083	1.6310	-0.27%	-0.06%	-0.58%	-0.3951	-0.0914	-0.7450	4.82E-05			
4525		1.0462	2.0047	1.6610	-0.05%	-0.18%	1.83%	-0.0674	-0.2666	2.3959	4.61E-05			
4526		1.0432	2.0269	1.6667	-0.29%	1.10%	0.34%	-0.4434	1.6716	0.4130	4.38E-05			
4527		1.0421	2.0355	1.6897	-0.10%	0.42%	1.37%	-0.1564	0.6224	1.6859	4.21E-05			
4528		1.0377	2.0257	1.7033	-0.42%	-0.49%	0.80%	-0.6707	-0.7238	0.9592	4.00E-05			
4529		1.0288	2.0246	1.6940	-0.87%	-0.05%	-0.55%	-1.3878	-0.0821	-0.6591	3.89E-05			
4530		1.0260	2.0189	1.6872	-0.27%	-0.28%	-0.40%	-0.4254	-0.4315	-0.4861	4.08E-05			
4531		1.0261	2.0126	1.6840	0.01%	-0.31%	-0.19%	0.0173	-0.4791	-0.2393	3.91E-05			
4532		1.0314	2.0001	1.6895	0.52%	-0.62%	0.32%	0.8535	-0.9826	0.4130	3.71E-05			
4533		1.0406	1.9905	1.6758	0.89%	-0.48%	-0.81%	1.4631	-0.7591	-1.0591	3.66E-05			
4534		1.0444	2.0022	1.6769	0.36%	0.58%	0.06%	0.5802	0.9288	0.0837	3.88E-05			
4535		1.0440	2.0118	1.6797	-0.03%	0.48%	0.17%	-0.0518	0.7593	0.2238	3.75E-05			
4536		1.0430	1.9867	1.6940	-0.09%	-1.26%	0.85%	-0.1583	-2.0162	1.1614	3.56E-05			
4537		1.0551	2.0060	1.6823	1.15%	0.97%	-0.69%	1.9808	1.4760	-0.9416	3.39E-05			
4538		1.0535	2.0117	1.6730	-0.15%	0.28%	-0.56%	-0.2470	0.4243	-0.7634	3.90E-05			
4539		1.0461	2.0086	1.6765	-0.71%	-0.16%	0.21%	-1.1586	-0.2353	0.2882	3.71E-05			
4540	Apr-08	1.0461	2.0001	1.6754	0.00%	-0.43%	-0.06%	0.0000	-0.6592	-0.0880	3.78E-05			

Figure 18: Excel model for U.S. dollar GARCH forecasting

5.2 Filtered Historical Simulation For Funds

As the FHS for returns sampled a set of past standardized residuals, so does the FHS for funds sample the set of past residuals specified by each Autobox model. For example, let $\{\hat{Z}_{t+1-\tau} : \tau = 1, 2, \ldots, M\}$ be the set of past residuals for the USD operational budget fund where the residual at time t is defined by equation (13) to be

$$\begin{aligned}
\varepsilon_t &= y_t - 0.4440\, y_{t-1} - 0.5010\, y_{t-12} + 0.2220\, y_{t-13} - 0.4945\, y_{t-24} + 0.2196\, y_{t-25} \\
&- 0.5450 - [\mathcal{K}] \Big(35.0920\, x_{t=48} + 17.9884\, x_{t=55} + 16.6461\, x_{t=63} \\
&+ 10.4465\, x_{t\geq 92} - 11.4907\, x_{t=97} + 21.4907\, x_{t=104} \\
&+ 22.3313\, x_{t=108,120,132,\ldots} + 64.9911\, x_{t=119} + 23.9756\, x_{t=120} \Big), \qquad (24)
\end{aligned}$$

where $[\mathcal{K}] = [1 - 0.4440B - 0.5010B^{12} + 0.2220B^{13} - 0.4945B^{24} + 0.2196B^{25}]$, as defined previously, act only on the intervention variables, x_t.

The process to determine the estimated expenditure is simpler then that for the returns as there are no intermediate calculations. Simply choosing a τ from $1, 2, \ldots, M$ will yield the current residual as input to equation (13) for the estimated expenditure, where all other values are found as linear combinations of past expenditures and intervention variables. Also, rather then calculating the expenditure on a daily basis, since the set of $\hat{\varepsilon}_{t+1-\tau}$ is based on monthly data, the calculation of expenditures is also done monthly for each iteration. Therefore, for the next 264 trading days, a fund expenditure is matched to an exchange rate as in Figure 17, i.e., every 22 trading days. Figure 19 describes the process whose end result is 10,000 sequences of hypothetical expenditures for day $t+22$, $t+44$, $\ldots$, $t+264$.

Figure 19: The FHS process for fund expenditures

Iterations $\downarrow$ **Days** $\longrightarrow$

$$\left\{ \begin{array}{cccc}
\hat{\varepsilon}_{1,22} \to \hat{y}_{1,22} & \hat{\varepsilon}_{1,44} \to \hat{y}_{1,44} & \cdots & \hat{\varepsilon}_{1,264} \to \hat{y}_{1,264} \\
\hat{\varepsilon}_{2,22} \to \hat{y}_{2,22} & \hat{\varepsilon}_{2,44} \to \hat{y}_{2,44} & \cdots & \hat{\varepsilon}_{2,264} \to \hat{y}_{2,264} \\
\vdots & \vdots & \vdots & \vdots \\
\vdots & \vdots & \vdots & \vdots \\
\hat{\varepsilon}_{10k,22} \to \hat{y}_{10k,22} & \hat{\varepsilon}_{10k,44} \to \hat{y}_{10k,44} & \cdots & \hat{\varepsilon}_{10k,264} \to \hat{y}_{10k,264}
\end{array} \right.$$

$$\begin{array}{cccc}
\downarrow & \downarrow & \cdots & \downarrow \\
\text{VaR} & \text{VaR} & & \text{VaR}
\end{array}$$

5.2.1 The Excel Model for Fund Expenditures

The above section is also prototype modelled in Excel and a sample of the main USD operational budget worksheet is shown in Figure 20. While the actual historical data goes from row 2 to row 121 (120 monthly expenditure values), the sample shown cuts off at row 5 and continues at row 97. The dashed red line at row 121 signifies the division between actual and forecasted values. Therefore, rows 122 through 133 show the sampled forecasted monthly results from April 2008 through March 2009, with the expenditure at the end of the month (highlighted in yellow) extracted for the VaR calculation as shown in Figure 19.

There are 14 columns in Figure 18 labelled A through N. From row 2 through 121:

- Column A displays the number for each data point, $t = 1, \ldots, 120$.
- Column B displays the month and year for which the fund data is aggregated.
- Column C displays the actual monthly expenditure for the U.S. dollar operational budget fund.
- Column D displays the value in Column C in millions of dollars (working with small numbers is preferable for this type of modelling).
- Column E displays the residual, ε_t, specified by equation (24). Since the largest lag is 25 months, the residual and model fit calculations necessarily start at $t = 26$.
- Columns F through N display the interventions as specified by Autobox, i.e.,
  1. Single Pulse at $t = 119$ of magnitude +64.9911;
  2. Single Pulse at $t = 48$ (not shown) of magnitude +35.0920;
  3. Single Pulse at $t = 104$ of magnitude +21.0530;
  4. Seasonal Pulse starting at $t = 108$ of magnitude +22.3313;
  5. Single Pulse at $t = 55$ (not shown) of magnitude +17.9884;
  6. Level Shift starting at $t = 92$ of magnitude +10.4465;
  7. Single Pulse at $t = 63$ (not shown) of magnitude +16.6461;
  8. Single Pulse at $t = 97$ of magnitude -11.4907;
  9. Single Pulse at $t = 120$ of magnitude +23.9756.

The forecasting portion of Figure 20 (from row 122) starts by first drawing with replacement from the set of past residuals, $E29 : E121$, through sampling a discrete uniform distribution of elements. The forecasted expenditure (highlighted) is then calculated through equation (13).

	A	B	C	D	E	F	G	H	I	J	K	L	M	N
1			USDOpBudget	OpBudget/10^6	ε_t	1	2	3	4	5	6	7	8	9
2	1	Apr-98	841840.04	0.8418										
3	2	May-98	3414998.77	3.4150										
4	3	Jun-98	2509636.75	2.5096										
5	4	Jul-98	3158109.58	3.1581										
97	96	Mar-06	53265904.65	53.2659	10.97690						10.4465			
98	97	Apr-06	3915382.03	3.9154	-6.86826						10.4465		-11.4907	
99	98	May-06	17415343.28	17.4153	-1.75503						10.4465			
100	99	Jun-06	9500816.69	9.5008	-6.06374						10.4465			
101	100	Jul-06	18695826.69	18.6958	1.08507						10.4465			
102	101	Aug-06	17790255.89	17.7903	-3.35374						10.4465			
103	102	Sep-06	16562101.14	16.5621	0.74034						10.4465			
104	103	Oct-06	22226025.73	22.2260	7.32486						10.4465			
105	104	Nov-06	45548768.52	45.5488	8.65006			21.053			10.4465			
106	105	Dec-06	33089044.66	33.0890	7.22870						10.4465			
107	106	Jan-07	40568441.71	40.5684	14.12346						10.4465			
108	107	Feb-07	37929524.66	37.9295	8.70472						10.4465			
109	108	Mar-07	75464893.96	75.4649	-4.17895				22.3313		10.4465			
110	109	Apr-07	9060939.92	9.0609	-9.53615						10.4465			
111	110	May-07	21693469.64	21.6935	6.03489						10.4465			
112	111	Jun-07	10204056.44	10.2041	-4.01166						10.4465			
113	112	Jul-07	14198614.2	14.1986	-7.47604						10.4465			
114	113	Aug-07	14227479.39	14.2275	-1.25400						10.4465			
115	114	Sep-07	14411519.17	14.4115	-1.31454						10.4465			
116	115	Oct-07	16723850.3	16.7239	-1.78612						10.4465			
117	116	Nov-07	21955937.53	21.9559	5.12115						10.4465			
118	117	Dec-07	17164800.46	17.1648	-12.87211						10.4465			
119	118	Jan-08	26730606.45	26.7306	0.38471						10.4465			
120	119	Feb-08	93704996.52	93.7050	0.05256	64.9911					10.4465			
121	120	Mar-08	99494500.06	99.4945	0.06712				22.3313		10.4465			23.9756
122	121	Apr-08		10.55787	-2.2893						10.4465			
123	122	May-08		17.90062	-1.4396						10.4465			
124	123	Jun-08		10.78508	1.1002						10.4465			
125	124	Jul-08		15.10140	-2.2610						10.4465			
126	125	Aug-08		23.78342	7.8439						10.4465			
127	126	Sep-08		15.66830	-3.8034						10.4465			
128	127	Oct-08		20.38797	0.3313						10.4465			
129	128	Nov-08		23.83008	-0.3087						10.4465			
130	129	Dec-08		24.60210	-1.2540						10.4465			
131	130	Jan-09		34.60680	0.7403						10.4465			
132	131	Feb-09		36.25641	2.0253						10.4465			
133	132	Mar-09		78.31335	1.0995				22.3313		10.4465			

Figure 20: Excel model for U.S. dollar Operational Budget fund forecasting

5.3 Building the VaR Model

In section 3, fund models were built as linear combinations of past expenditures, intervention variables and current values of white noise disturbance terms. Changing the notation slightly to fit equation (1), the forecast expenditures are given functionally as

$$E^k_{c,a,t+22n} = f_{c,a}(\varepsilon_{t+22n},\, \phi_j y_{t-j}) \,, \tag{25}$$

where the subscripts c, a denote the currency and account (or fund) respectively; $k = 1, \ldots, 10,000$, the number of iterations in the FHS process; $n = 1, \ldots, 12$, the number of months; $j = 1, \ldots, p$, the number of autoregressive terms respectively, with some ϕ taking on zero values.

Similarly, based on the results of section 4, the forecasted exchange rates can be written functionally as

$$p^k_{c,t+22n} = f_c(\hat{z}_{t+22n},\, \sigma_{t+22n},\, r_{t+22n}) \,, \tag{26}$$

where c, k and n were previously defined.

Given that the budget rates are also forecast on a monthly basis, but fixed by external sources, i.e., b_{t+22n}, we can write the relationship that defines the fund variance as a variation on equation (1)

$$\left\{ V^k_{c,a,n} = E^k_{c,a,t+22n} \times (b_{c,t+22n} - p^k_{c,t+22n}) \,, k = 1, \ldots, 10,000 \right\}^{12}_{n=1} \,, \tag{27}$$

where $V^k_{c,a,n}$ is the variance for currency c, account a, iteration k and month n, and b, the budget rate, is fixed for each n. The VaR is therefore defined by the 5th percentile of equation (27), i.e.,

$$VaR^{0.05}_{c,a,n} = \left\{ V^k_{c,a,n} \,, k = 1, \ldots, 10,000 \right\}^{0.05} \,, \tag{28}$$

for any n month.

6 Simulation Results

The methodologies described in the preceding sections are combined into a risk simulation that uses filtered historical simulation with Latin Hypercube stratified sampling to ensure good representation of actual variability.

The simulation forecasts per month for a 12-month period starting 01 April 2008. Each fund account is forecasted per month for the following 12 months using a uniform distribution to sample the expenditure residuals set as shown in Figures 19 and 20. Each currency return is forecasted per day for the following 12 months (matching expenditure 12 month period) using a uniform distribution to sample the set of standardized returns as shown in Figures 16 and 18. For every 22nd trading day, the forecasted exchange rate is extracted to produce the variance through equation (27) and ultimately the VaR through equation (28).

6.1 Forecasting Expenditures

The simulation was run for 10,000 iterations. The expenditures per month for four months ahead (relative to March 2008) are given in Table 12 for U.S. dollar funds, partitioned by 0th (minimum expenditure), 5th, 50th (median), 95th and 100th (maximum expenditure) percentiles of a distribution of 10,000 sequences based on the algorithm depicted in Figure 19. Comparing Table 12 with Table 4, we see that only those models with an autoregressive structure (L101, L501, C113, C107, C160 and Op Budget) describe forecast variability. As opposed to the remaining models whose future expenditures are described by a constant and a fixed intervention structure, the AR components factor the past history into the forecast to yield a more robust structure. It stands to reason, however, that with time and more USD transactions, particularly for the ‘V’ funds, a more equitable model structure will be developed for: L518, C503, V511, V510, C001 and their roll-ups.

Table 12: Expenditure percentile forecast results for U.S. dollar funds

Months	L101	L501	L518	C503	C113	V511	V510	C001	C107	C160	Op. Budget	Invest. Cash	Other
Zeroth percentile (minimum) expenditure													
Apr-08	0	0	140,101	0	0	1,671	737	0	0	42,397	0	1,671	0
May-08	352,744	0	140,101	0	0	1,671	737	0	0	35,457	3,004,572	1,671	0
Jun-08	0	0	140,101	0	0	1,671	737	0	0	86,299	0	1,671	0
Jul-08	0	0	140,101	0	0	1,671	737	0	0	66,620	0	1,671	0
Fifth percentile expenditure													
Apr-08	18,822	0	187,732	5,107,231	2,732,319	1,671	737	0	0	92,503	5,978,965	1,671	0
May-08	10,883,409	481,340	187,732	5,536,453	3,528,429	1,671	737	0	0	85,563	12,955,644	1,671	0
Jun-08	1,038,464	0	187,732	5,107,231	2,971,122	1,671	737	0	0	136,405	3,146,679	1,671	0
Jul-08	3,057,248	0	187,732	5,107,231	2,052,918	1,671	737	0	0	116,726	9,759,815	1,671	0
50th percentile expenditure													
Apr-08	6,626,416	148,244	489,101	19,981,356	9,105,309	287,914	3,246	0	0	428,715	12,667,631	20,001,508	1,264,604
May-08	18,150,248	1,643,774	485,345	19,981,356	10,673,346	287,914	3,246	0	172,303	421,775	20,185,406	20,001,508	1,214,833
Jun-08	8,825,802	693,754	489,101	20,258,456	10,171,096	287,914	3,246	0	75,038	472,617	10,642,535	26,794,260	1,214,833
Jul-08	11,282,051	1,074,234	485,345	20,258,456	9,171,236	19,998,262	3,246	0	237,757	452,938	17,187,006	26,794,260	1,214,833
95th percentile expenditure													
Apr-08	13,112,778	1,937,197	982,510	55,794,468	17,736,244	142,212,000	361,388	4,905,290	591,624	828,003	21,497,284	142,212,000	5,936,800
May-08	26,113,630	3,434,060	982,510	55,507,960	19,890,316	142,212,000	361,388	4,905,290	755,064	821,062	29,091,242	142,212,000	5,936,800
Jun-08	17,568,004	2,484,040	982,510	55,794,468	19,311,404	142,212,000	361,388	4,905,290	659,857	871,904	19,497,490	142,212,000	5,936,800
Jul-08	20,317,174	2,863,188	982,510	55,794,468	18,522,978	142,212,000	361,388	4,905,290	817,979	852,225	26,372,114	142,212,000	5,936,800
100th percentile (maximum) expenditure													
Apr-08	18,564,284	2,895,390	1,126,041	70,420,648	21,569,176	142,212,000	361,388	6,998,409	1,277,567	1,137,966	26,970,684	142,212,000	8,254,465
May-08	38,623,372	4,392,253	1,126,041	70,420,648	26,671,642	142,212,000	361,388	6,998,409	1,759,375	1,131,025	39,027,236	142,212,000	8,254,465
Jun-08	29,062,488	3,442,233	1,126,041	70,420,648	26,983,252	142,212,000	361,388	6,998,409	1,679,411	1,181,867	32,594,666	142,212,000	8,254,465
Jul-08	35,433,280	3,821,381	1,126,041	70,420,648	25,522,676	142,212,000	361,388	6,998,409	1,651,629	1,162,188	38,022,784	142,212,000	8,254,465

6.1.1 Forecasted expenditure validation

Notwithstanding the small sample size for a number of funds, Table 13 displays the results of ex-ante, “out-of-sample”, testing of expenditure forecasting accuracy. In other words, monthly data prior to April 2008 was used to fit the model (the fit period), and monthly data post March 2008 (the test period) was reserved to assess the model’s forecasting accuracy. For each actual expenditure, the corresponding forecasted percentile was interpolated from the forecasted expenditure cumulative distributions.

Inspection of Table 13 shows, for most funds, the actuals are randomly distributed about the median. For capital expenditures (C503), randomness is also experienced, however, the very nature of capital introduces a complexity to the model. The annual (1 April - 31 March) capital spending pattern is observed to be non-linear with increasing trend in the monthly frequency of payments and their corresponding magnitude as the fiscal year progresses. This occurs because capital contracts are of a fixed duration often with flexible payment and delivery schedules. It is observed that large payments occur in the final quarter of the fiscal year leaving a significantly smaller payment for the first quarter of the new fiscal year as the cycle repeats itself. For USD C503, for example, Autobox forecasted a model with no AR components, but two seasonal pulses of period 12 starting March 2001, and a level shift of magnitude +13.17, which together with the constant value specified a forecast mean of +22.72[20] with 5th and 95th percentile values at 0.0 and 53.29 respectively. The actuals specified in Table 13 are significantly below the mean but consistent with previous values in the same periods.

Figure 21 illustrates the cumulative distribution of expenditures for USD forecasted operational budget transactions from April 2008 (Figure 21a) through July 2008 (Figure 21d). Also shown is the actual expenditure value for each month as well as their percentiles. While the distributions that the results are drawn from are not excessively skewed, each does exhibit fairly high kurtosis relative to normal, i.e., > 4.4.

The operational budget fund is a roll-up of three funds, L101, L501 and L518, of which L101, being an order of magnitude greater than the other two, defines the structure of the overall fund. Therefore, any forecasting issues with L101 will necessarily translate into issues for the operational budget fund. In Figure 21 we note that actual values for May – July 2008 are found at the tail end of the distribution, and in the case of June, completely outside the distribution of possible forecasts. Concurrently, the maximum possible values for May – July 2008 for L101 were found to be 38.6, 28.6 and 31.4 respectively, and therefore actual values for the same period (see Table 13) of 34.3, 33.5 and 24.0 respectively, are also to be found at the tail end of the distribution or, as in the case of June, external to the spread. Clearly, the latest values are inconsistent with expectations founded on 10 years of past data and could not be forecasted. There appears to be a new trend forming starting April 2008 which, if better understood through studying the causal events, could be predicted through incorporating a new predictor variable or redesigning the model over time.

[20] All values are in millions of dollars CAD

Table 13: Results of interpolation of actual expenditures to the forecasted distribution; Funds in red need to be redesigned to incorporate new trends

Fund	April 2008		May 2008		June 2008		July 2008	
	Actual Value	Perc.	Actual Value	Perc.	Actual Value	Perc.	Actual Value	Perc.
L101	10,434,619	85	34,328,395	100	33,469,192	100	24,004,231	99
L501	33,906	44	3,106,197	94	1,780,562	82	839,560	38
L518	1,771,669	100	1,750,856	100	3,780,304	100	1,077,060	98
C503	2,921,856	2	10,473,685	20	6,731,278	10	6,709,462	10
C113	4,685,690	21	19,741,075	95	4,576,776	11	8,873,407	48
V511	76,584,511	93	45,270	14	0	0	3,660,946	50
V510	0	0	1,639	22	5,658	56	34,113	56
C001	0	55	0	55	0	55	0	55
C107	24,466	54	28,499	31	33,288	44	182,596	42
C160	482,134	70	74,254	4	249,281	19	219,805	19
Op Budget	12,240,194	42	39,185,448	100	39,030,058	100	25,920,851	95
Invest. Cash	76,584,511	93	46,908	7	5,658	7	3,695,059	36
Other	506,600	32	102,753	18	282,569	24	402,401	28

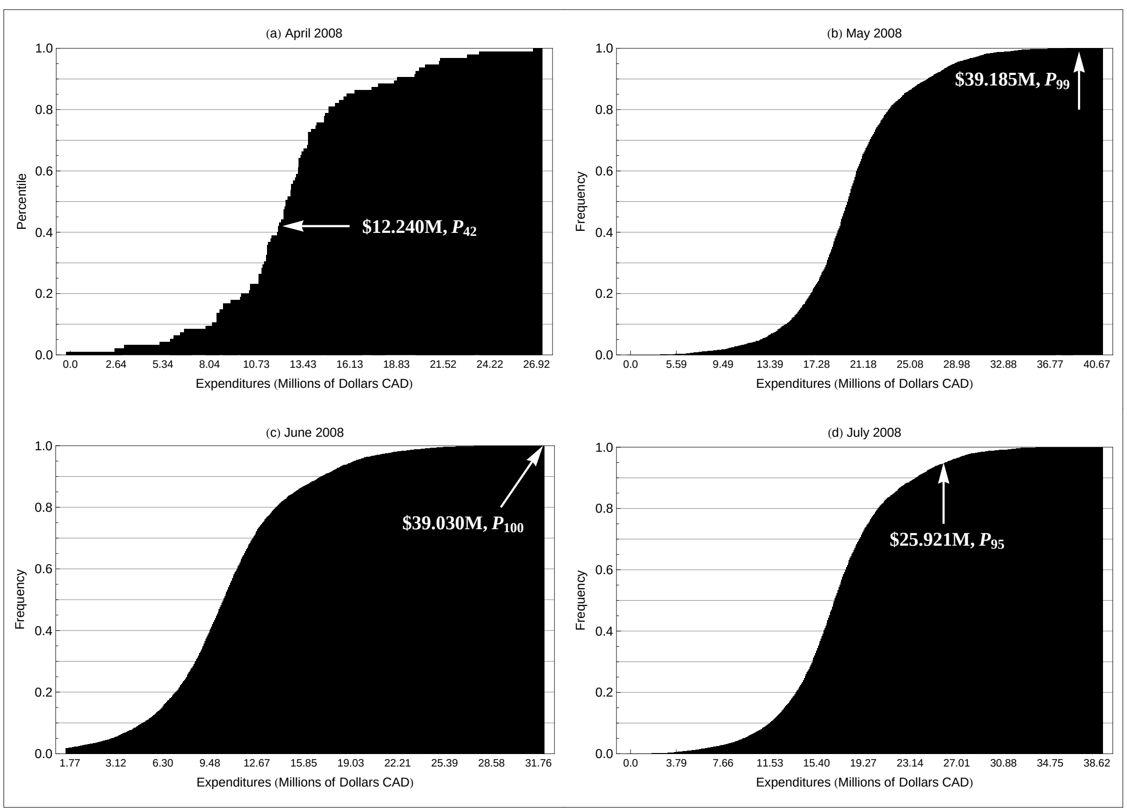

Figure 21: Cumulative expenditure distribution for USD operational budget fund from April 2008 – July 2008; Actual values and their percentiles are specified.

6.2 Forecasting Performance of Currency Returns

There is really no reliable method to forecast exchange rates and we have not attempted to do so here. Models for exchange rate movements are largely driven by changes in macroeconomic factors like unexpected economic or political events, interest rates, the pattern of trade between one country and another and what is known as absolute purchasing power parity (PPP) which holds that goods-market arbitrage will tend to move the exchange rate to equalize prices between countries ([38]).

Currently, DND uses time series methods for short-term prediction of exchange rates ([39]). Simple ARIMA models attempt to isolate trends in past data to predict future values. While much simpler then economic models that rely on explanatory variables, they only rely on past data and ignore causal relations that influence future expectations.

The VaR model in this study was meant to forecast expected foreign exchange risk and not expected returns. Nevertheless, in calculating the VaR from equations (27, 28), a return distribution from the FHS process is given as a product of the sampled standardized return and the modelled GARCH variance as in equation (20). Figures 22 – 24 illustrate the return distribution of each currency return forecasted one month ahead from 31 March 2008. Note the higher peak of CAD/USD as originally specified through the excess kurtosis in Table 10. Table 15 displays the ex-ante testing of return forecasting accuracy. Actual returns were calculated by applying the log rate change to the Bank of Canada rates for end-of-months: April-July 2008 inclusive ([19]). For each actual return, the corresponding percentile was interpolated from the forecasted returns distribution. For example, the data for Figures 22 – 24 would be used to interpolate the one-month ahead percentile from the actual value.

Although the actuals are reasonably close to the median, Table 15 nevertheless shows the actual rates to be distributed to the left of the median rather than randomly on both sides. Should the trend continue, the GARCH models for each currency would need to be examined in greater detail to ensure volatility is correctly accounted for and that a bias towards underforecasting the rate hasn't materialized in the calculations.

Table 14: Exchange Rate percentile forecast results

Months	USD	GBP	EUR
Zeroth percentile (minimum) rate			
Apr-08	0.7907	1.7030	1.3860
May-08	0.7017	1.6888	1.3224
Jun-08	0.6239	1.6284	1.2216
Jul-08	0.6009	1.5714	1.1646
Fifth percentile rate			
Apr-08	0.9705	1.9340	1.5351
May-08	0.9485	1.8962	1.5001
Jun-08	0.9310	1.8699	1.4726
Jul-08	0.9166	1.8490	1.4536
50th percentile rate			
Apr-08	1.0263	2.0429	1.6221
May-08	1.0270	2.0454	1.6191
Jun-08	1.0271	2.0491	1.6172
Jul-08	1.0270	2.0515	1.6154
95th percentile rate			
Apr-08	1.0881	2.1591	1.7191
May-08	1.1138	2.2088	1.7537
Jun-08	1.1341	2.2458	1.7829
Jul-08	1.1502	2.2796	1.8052
100th percentile (maximum) rate			
Apr-08	1.2880	2.4186	1.8597
May-08	1.4234	2.5506	2.0938
Jun-08	1.6058	2.6483	2.0888
Jul-08	1.8365	2.7336	2.2079

Table 15: Results of interpolation of actual returns to the forecasted cumulative distribution

Months	CAD/USD		CAD/GBP		CAD/EUR	
Ahead	Actual Value	Perc.	Actual Value	Perc.	Actual Value	Perc.
Apr-08	1.0072	28	2.0034	27	1.5714	18
May-08	0.9930	23	1.9676	19	1.5468	16
Jun-08	1.0197	45	2.0276	42	1.6041	44
Jul-08	1.0240	48	2.0312	44	1.5993	44

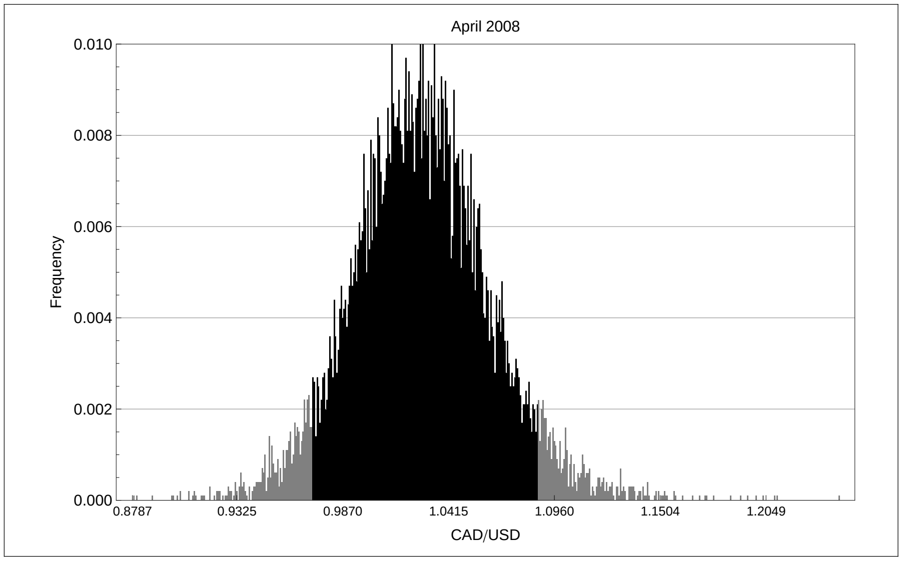

Figure 22: Return Distributions for CAD/USD exchange for one month ahead from 31 March 2008. Shaded areas to left and right of average correspond to the lower and upper 5% of results respectively.

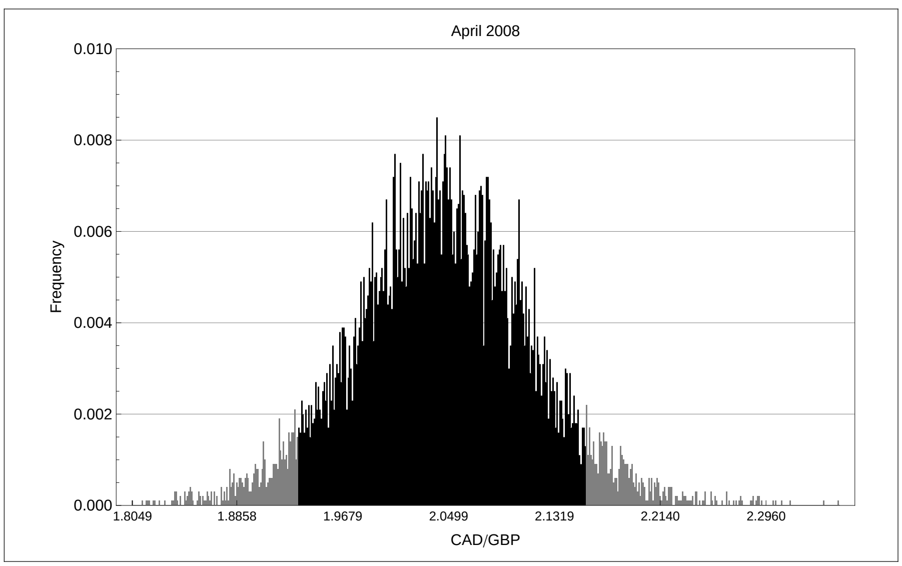

Figure 23: Return Distributions for CAD/GBP exchange for one month ahead from 31 March 2008. Shaded areas to left and right of average correspond to the lower and upper 5% of results respectively.

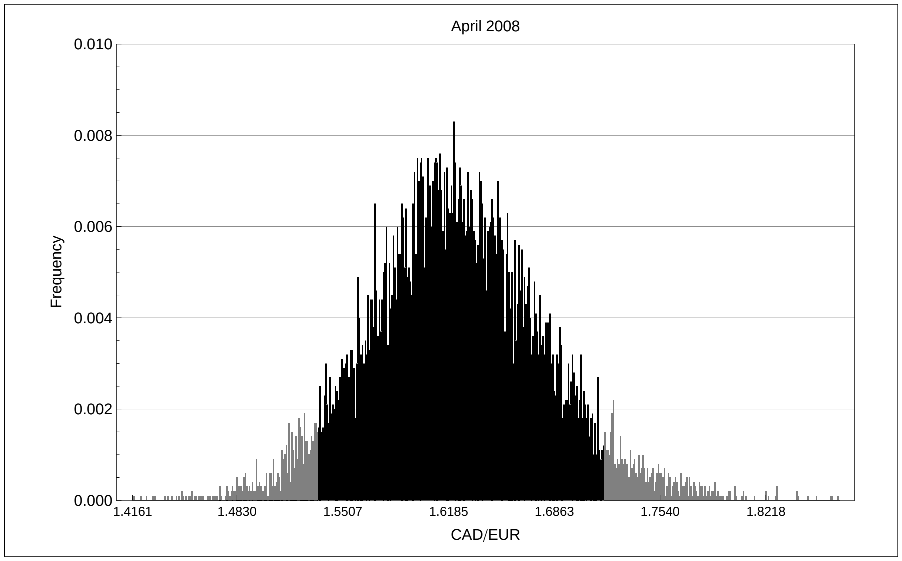

Figure 24: Return Distributions for CAD/EUR exchange for one month ahead from 31 March 2008. Shaded areas to left and right of average correspond to the lower and upper 5% of results respectively.

6.3 Forecasting Variance and Value-at-Risk

Table 16 gives the DND budget rates (*b*) for equation (27). The variance results per month for four months ahead (relative to March 2008) are given in Table 17, partitioned by 5th (VaR), 50th (median) and zeroth (maximum expected loss) percentiles of a distribution of 10,000 sequences of equation (27). For example, Figure 25 illustrates the output for CAD/USD forecasted operational budget transactions for April 2008 – July 2008 inclusive. The shaded areas to the left and right of average correspond to the lower and upper 5% of the results respectively. Since we are mainly interested in the VaR, the value at the 5th percentile is reported in the upper portion of Table 17. The median (50th percentile) of the distribution, which could be a loss or a gain, is reported in the middle portion of the table. Values close to zero imply a budget rate that is close to the forecasted exchange rate. The maximum expected loss (0th percentile) is reported at the bottom of the table and is reflective of significant differences between the budget rate and the forecasted exchange rate.

Figure 25 plots the entire variance distribution for each month and shows that each distribution is skewed left with a long tail that is sparsely populated. Clearly extreme values can be reported as, unlike historical simulation, FHS can forecast large losses even if a large loss was never recorded in the historical data set.

The sharp peaks for April and June are unique to this type of analysis and are reflective of the difference calculation in the variance equation (27) where b, the assigned budget rate, is equal to p, the forecasted exchange rate, i.e., the single peak contain the zeros of the variance equation. Single peaks are not found in the charts for May and July because the budget rates were found to be in the tails of the distribution and not around the median.

Table 16: DND forecasted budget rate

Months	USD	GBP	EUR
Apr-08	1.0139	2.0089	1.5972
May-08	0.9994	1.9653	1.5555
Jun-08	1.0125	1.9648	1.5757
Jul-08	1.0243	1.9679	1.5771

Table 17: Variance and Value-at-Risk forecasted percentile results for U.S. dollar funds

Months	L101	L501	L518	C503	C113	V511	V510	C001	C107	C160	Op. Budget	Invest. Cash	Other
5th percentile loss (Value-at-Risk)													
Apr-08	-577,654	-61,576	-41,926	-1,986,313	-793,377	-3,330,287	-6,757	-100,786	-17,739	-36,606	-1,005,811	-3,601,783	-189,519
May-08	-2,183,803	-235,185	-66,473	-3,187,971	-1,451,853	-5,550,985	-12,297	-178,959	-43,871	-58,129	-2,433,401	-5,825,957	-310,019
Jun-08	-1,260,627	-144,586	-68,635	-3,248,686	-1,431,951	-5,114,664	-10,516	-146,856	-34,506	-65,020	-1,458,758	-5,665,238	-296,685
Jul-08	-1,578,483	-184,070	-71,543	-3,286,016	-1,376,054	-4,932,777	-9,531	-125,907	-48,768	-63,823	-2,315,365	-5,449,278	-292,813
50th percentile gain/loss													
Apr-08	-56,974	0	-5,617	-184,257	-85,067	-1,314	-34	0	0	-3,625	-140,835	-2,059	-3,438
May-08	-465,289	-38,253	-12,237	-416,500	-239,520	-3,338	-80	0	-300	-7,994	-526,779	-5,871	-12,266
Jun-08	-75,805	-1,351	-6,325	-199,321	-105,506	-1,253	-38	0	0	-4,944	-113,314	-1,942	-2,382
Jul-08	-11,007	0	-1,074	-24,231	-4,559	-51	-6	0	0	-775	-37,852	-55	0
Zeroth percentile (expected maximum loss)													
Apr-08	-3,580,841	-628,555	-229,933	-12,027,102	-5,562,590	-29,202,416	-73,699	-1,279,706	-189,789	-196,084	-4,858,550	-29,202,416	-1,642,376
May-08	-10,448,332	-1,806,681	-651,169	-19,105,450	-7,436,613	-50,534,832	-114,370	-1,722,667	-578,966	-350,066	-12,679,790	-38,352,844	-2,058,718
Jun-08	-9,502,858	-1,218,545	-607,640	-23,071,172	-10,900,461	-90,019,000	-162,865	-2,327,150	-552,348	-385,296	-10,749,651	-55,104,280	-3,390,113
Jul-08	-14,778,071	-1,858,528	-1,064,413	-36,772,400	-9,005,898	-82,586,824	-366,072	-4,249,419	-637,456	-527,719	-22,602,636	-69,301,368	-4,475,938

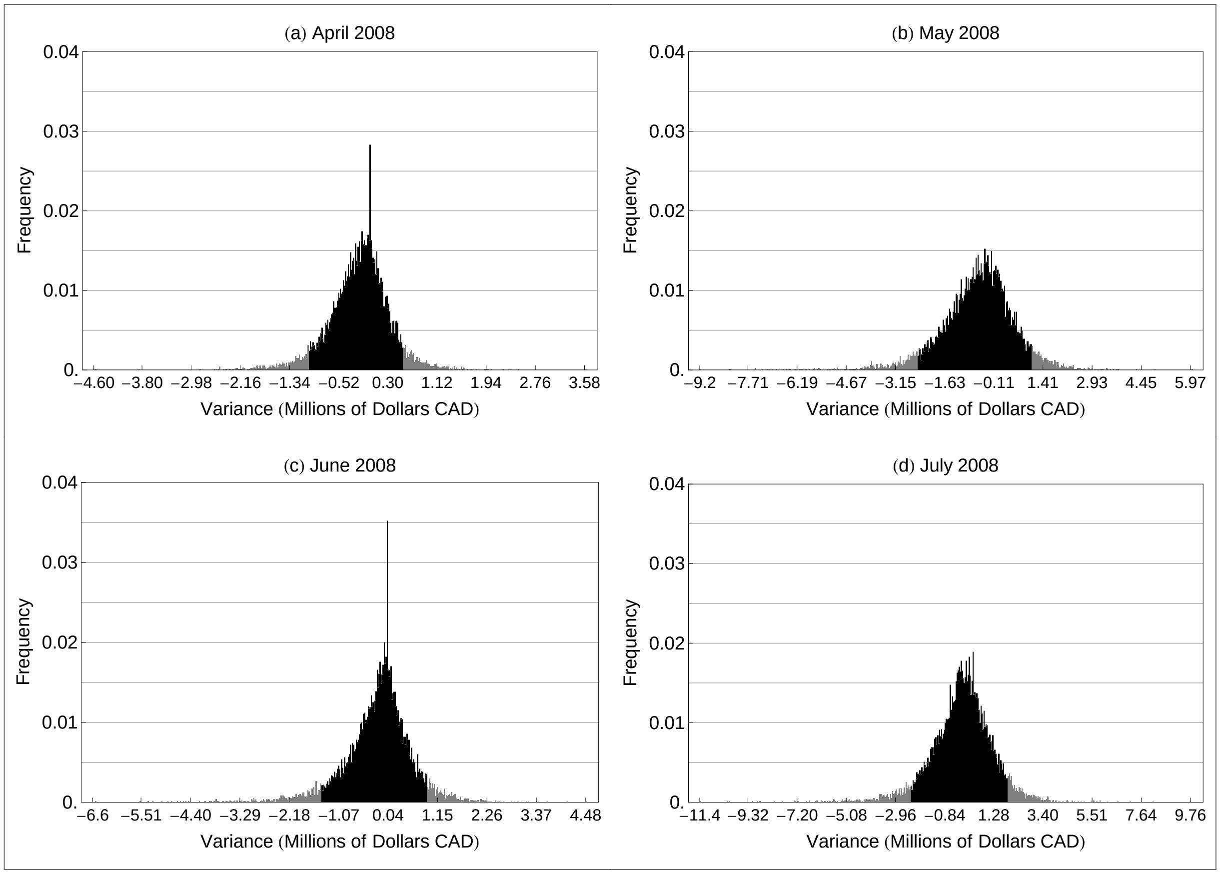

Figure 25: Variance forecasted distributions for CAD/USD operational budget fund from April 2008 through July 2008. Shaded areas to left and right of average correspond to the lower and upper 5% of results respectively.

6.3.1 Forecasted Variance Validation

The variance is defined by equation (27) and the Value-at-Risk taken (in this study) as the 5th percentile of the variance distribution. Since we know the actual fund expenditures and exchange rates for April – July 2008, the actual variance could also be calculated. Table 18 shows the actual variance for the specified periods as well as where the actuals fall within the VaR distributions (U.S. dollar distributions for the operational budget fund are shown in Figure 25.

The results of Table 18 provide a useful diagnostic of the VaR models for the funds. There are no observable trends in the percentiles.

Table 18: Results of interpolation of actual variance to the forecasted distribution

Fund	April 2008		May 2008		June 2008		July 2008	
	Actual Value	Perc.	Actual Value	Perc.	Actual Value	Perc.	Actual Value	Perc.
L101	69,912	78	218,672	81	-240,978	37	7,201	53
L501	227	80	19,786	82	-12,820	39	252	55
L518	11,870	86	11,153	86	-27,218	24	323	52
C503	19,576	67	66,717	76	-48,465	57	2,013	52
C113	31,394	70	125,751	82	-32,953	56	2,662	53
V511	513,116	89	288	76	0	60	1,098	60
V510	0	65	10	75	-41	49	10	54
C001	0	84	0	88	0	82	0	78
C107	164	84	182	81	-240	35	55	63
C160	3,230	76	473	74	-1,795	57	66	52
Op Budget	82,009	73	249,611	81	-281,016	39	7,776	52
Invest. Cash	513,116	87	299	75	-41	59	1,109	58
Other	3,394	75	655	76	-2,034	51	121	55

7 Future Development

From this point forward, all FOREX development for the department will be under contractor control with the author serving as project authority for development, and technical authority for the mathematical modelling component. ADM(Fin CS)/DSFC-7 will serve as the technical authority for the web application interface and output reports component. An Intranet, Defence Information Network (DIN) based application will be developed for the publication, presentation, and archival of the Value-at-Risk results. The web application will include expanded functionality including user roles, bilingual operations, and enhancements defined in the evaluation of the prototype.

Data will come from the following sources:

- Automatic Forecasting System /Autobox Application (updated expenditure coefficients);
- FMAS (current transactions);
- Bank of Canada (current exchange rates); and,
- DSFC (forecasted budget rates).

The output reports will project 3 months into the future, however, the capability to adjust the number of months will also exist. When a new report is published to the web, the old one is archived and stored for 2 years with access to the report restricted (username and password).

8 Conclusions

With the success of the original FOREX model, ADM(Fin CS) has a requirement to expand the model to include the two funds (national procurement and capital) analyzed in [1], plus eight additional funds that each account for over $10M in foreign transactions every year. This report documents the analysis and validation of the modelling required to calculate the risk of exposure to foreign exchange volatility over the budget year.

In this and in a previous studies [1, 2], we have developed financial expenditure models through Box-Jenkins mechanisms, albeit now automatically produced through the Autobox application; and, have modelled the conditional variances of the financial return series through the basic GARCH(1,1) model, where the GARCH weights have been specified by maximizing the log-likelihood of the standardized *t(d)* distribution for CAD/USD and CAD/GBP, and the normal distribution for CAD/EUR.

The individual models for expenditures and currencies were then combined into an overall departmental Value-at-Risk model. Results were then obtained through filtered historical simulation, which assumes no distributional assumptions but retains the non-parametric nature of the historical price change models by bootstrapping from the set of standardized residuals, which were standardized by the GARCH standard deviation.

Monthly forecasted expenditures were matched to exchange rates every 22 trading days to forecast a monthly variance. Simulating for 10,000 sequences of hypothetical daily returns, distributions were produced for expenditures, exchange rates and variances, and the results were validated through interpolating actual values and seeing how well they fit the distribution medians.

This study further illuminates certain policy implications for functional finance and performance/risk management specialists in the department. In particular, the VCDS Group through the Director Force Planning and Programme Coordination (DFPPC) and ADM(Fin CS) through Director Budget and Director Strategic Finance and Costing will want the capability to adjust corporate budget allocations (quarterly) based on the results of the FOREX model. Furthermore, these groups should consider adopting the VaR methodology as part of the department's integrated risk management framework for managing the budgetary risk attributed to exposure to foreign currency fluctuations for all acquisitions. Currently there is no tool available to assess the in-year impact of foreign exchange fluctuations on Defence budget allocations. FOREX will offer this capability.

By extension, the department should also examine opportunities to apply the VaR analytical approach to quantifying the financial risk in other budget expenditure areas subject to market/price risk such as bulk fuels, energy/hydro, and certain commodities (e.g., steel, ballistic materials, etc.) where expenditure amounts warrant. As the department embarks on large multi-year capital acquisitions and continues to be engaged in sizeable, complex overseas deployments, the need to measure and accurately assess financial risk has never been greater. Moreover, should the department decide to seek central government agency concurrence to implement (or pilot) a financial hedging strategy to limit foreign exchange risk (as is the case in the UK and proposed by Essaddam et al. [40]), the ability to measure and report exchange rate risk would be fundamental for successful hedging with forward contracts, futures or options[21]. A forward contract would protect the department should the exchange rate depreciate, but on the other hand, the advantage of a favourable exchange rate movement would have to be foregone. Hedging with futures is similar to forwards but is more liquid because it is traded in an organized exchange – the futures market. Currency options provide an insurance against falling below the strike price or the exercise price. However, because options are much more flexible compared to forwards or futures, they are also more expensive.

It remains to be seen if DND's unique requirements could best be served through a combination of options, futures and/or forward contracts. Notwithstanding, this study does illustrate the practical application of the VaR method to arguably the largest department financial risk area, foreign currency exposure, and it is hoped that it will contribute to a better understanding of this risk parameter and how it can be more consistently and accurately measured, reported and ultimately controlled through analysis.

[21] A forward contract is an agreement between two parties to buy or sell an asset for a fixed rate and at a specified point of time in the future. A futures contract gives the holder the obligation to make or take delivery under the terms of the contract but is exchange-traded, while forward contracts are traded over-the-counter. An option is a contract written by a seller that conveys to the buyer the right - but not the obligation - to buy or to sell a particular asset[41].

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Annex A: Exchange Rates and Canadian Dollar Variance for GBP and EUR Expenditure Categories

Section 2.2 discusses the basic relationship, equation (1), that defines all VaR and variance calculations for this study. This annex compares the budget rate against the liquidated rate for the GBP and EUR currencies and the five major expenditure categories: Operating Budgets, Capital (Equipment), National Procurement, Investment Cash and the miscellaneous category, Other account.

A.1 The GBP Rates and Variances

As also stated previously, capital (equipment) transactions can be an order of magnitude above operational budget transactions. Consequently, even small differences between the two exchange rates can mean large variances. Unfortunately, the annual budget forecasts for FY's 98/99 and 04/05 in Figure A.1 did not account for the dramatic increase in the actual exchange rate and the consequence being a large negative variances in both the GBP Capital and Op Budget transactions.

As far as NP is concerned (Figure A.2), there was a large \$20.4M transaction in period 13 of FY 02/03 for the submarine project, that was rolled-up with an excess of \$10M in transactions in period 12. Therefore, even with a small, 3.8%, difference in the exchange rates, the variance was still approximately +\$1.27M.

In the case of the Other funds for the period FY 06/07 (Figure A.3), while the change in variance is fairly dramatic, the magnitudes of the changes are not so excessive that they couldn't be absorbed within the local budgets.

A.2 The EUR Rates and Variances

The euro became an official currency on 01 January 1999, however it was not forecasted in the DND economic model prior to April 01, 1999. In any case, there were no transactions regarding the euro prior to December 1999. Two large transactions (\$9.4M and \$7.8M) in December 2002 were the cause of the large negative Capital variance shown in Figure A.4. The only other issues were the relatively large negative variance for Op Budget (-\$1.4M), Investment Cash (-\$3.8M) and Other (-\$2.9M) categories all found in period 12 of FY 07/08. The Op Budget variance could be explained by large L101 transactions (\$18.5M in period 12 and \$4.4M in period 13) for Op Athena. For Investment Cash there was one \$71.1M and a number of significantly smaller (but still large) transactions in period 13 for the armoured vehicle program; and, for the Other funds, there were \$57.7M in Grant & Contributions in period 12 acted upon by approximately a 5% difference in rates.

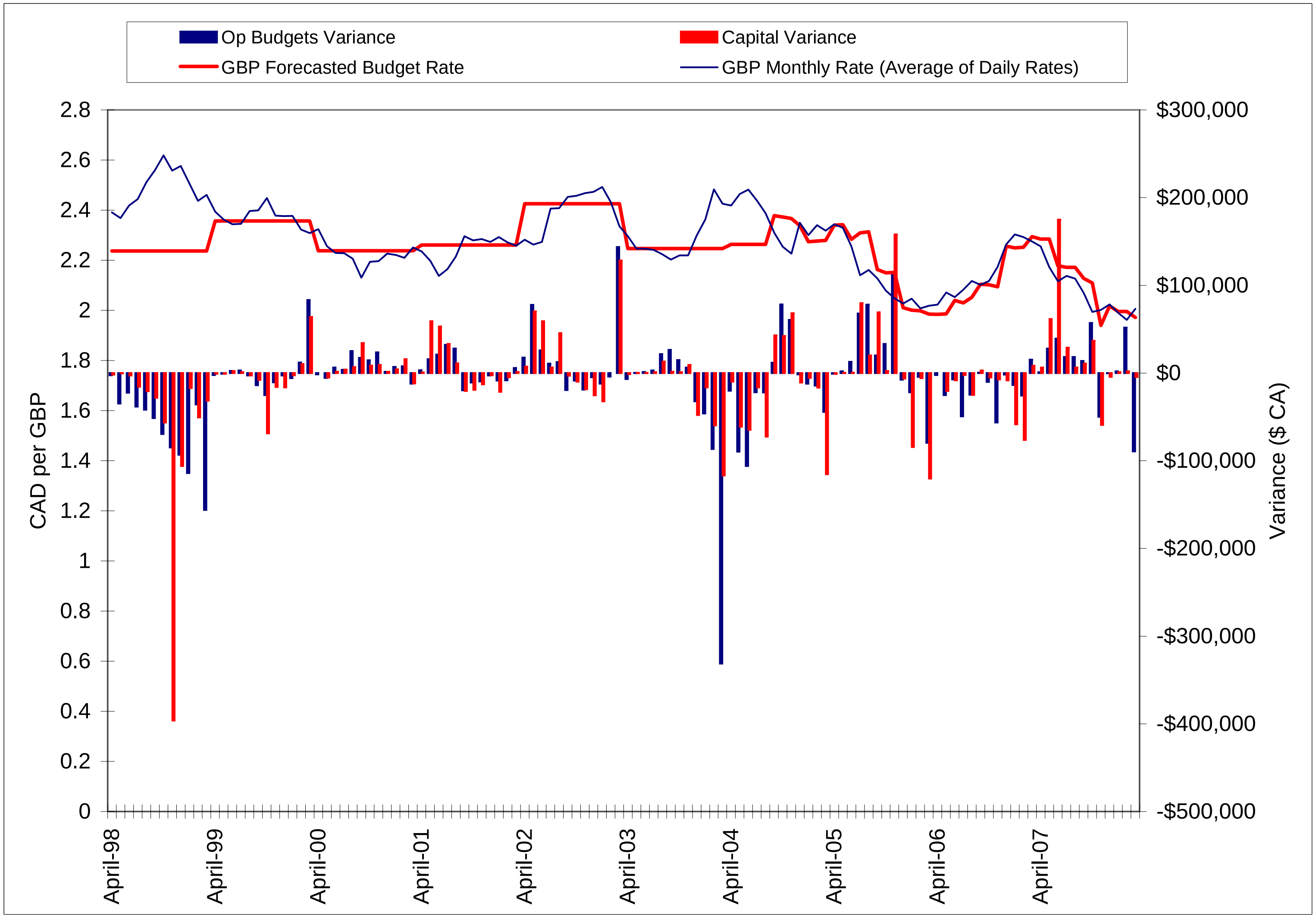

Figure A.1: Rates and Canadian dollar variance on U.K. sterling liquidated obligations (Operating Budget and Capital (equipment) categories). Left-hand scale shows exchange rate; Right-hand scale shows variance.

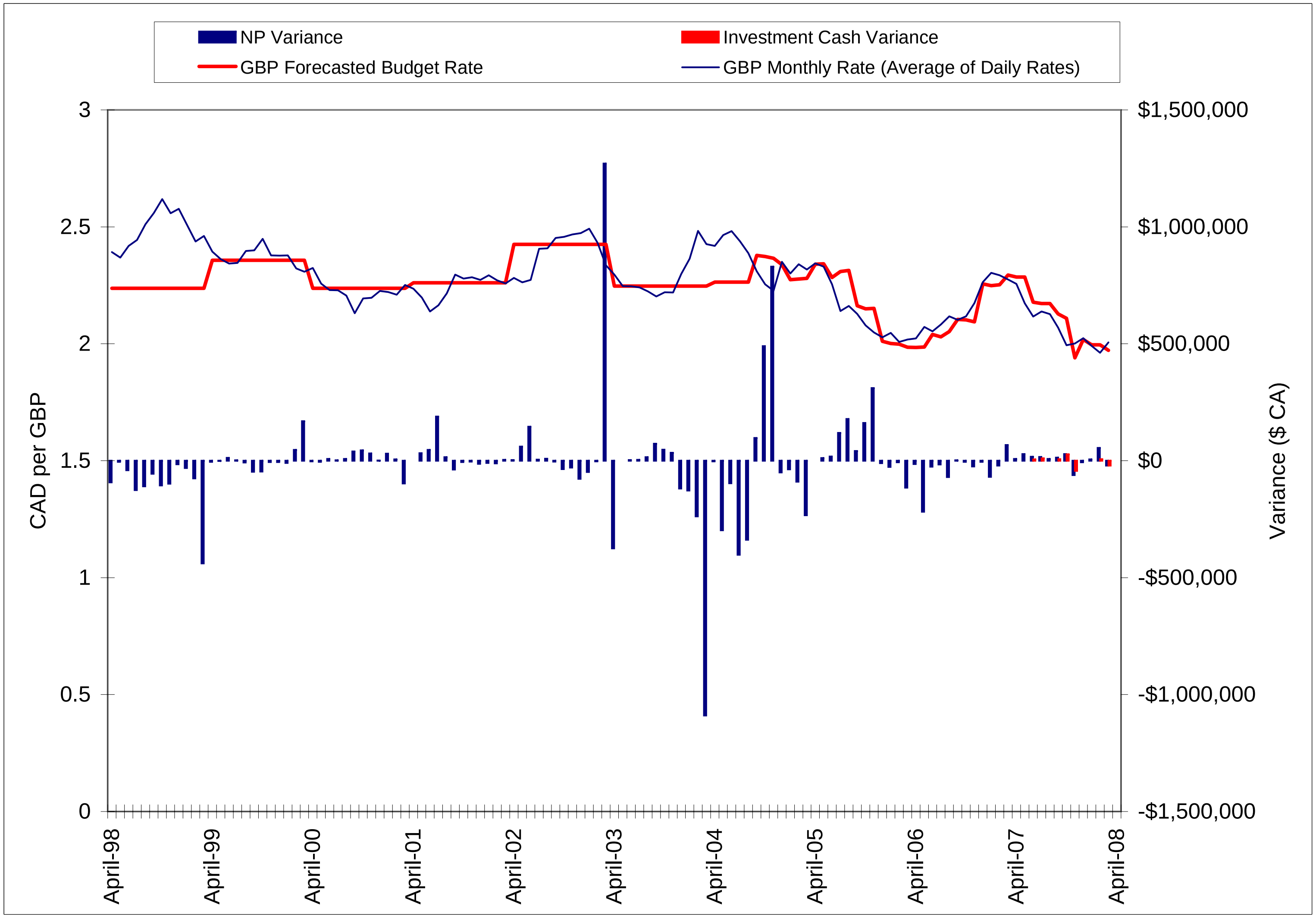

Figure A.2: Rates and Canadian dollar variance on U.K. sterling liquidated obligations (National Procurement and Investment Cash categories). Left-hand scale shows exchange rate; Right-hand scale shows variance.

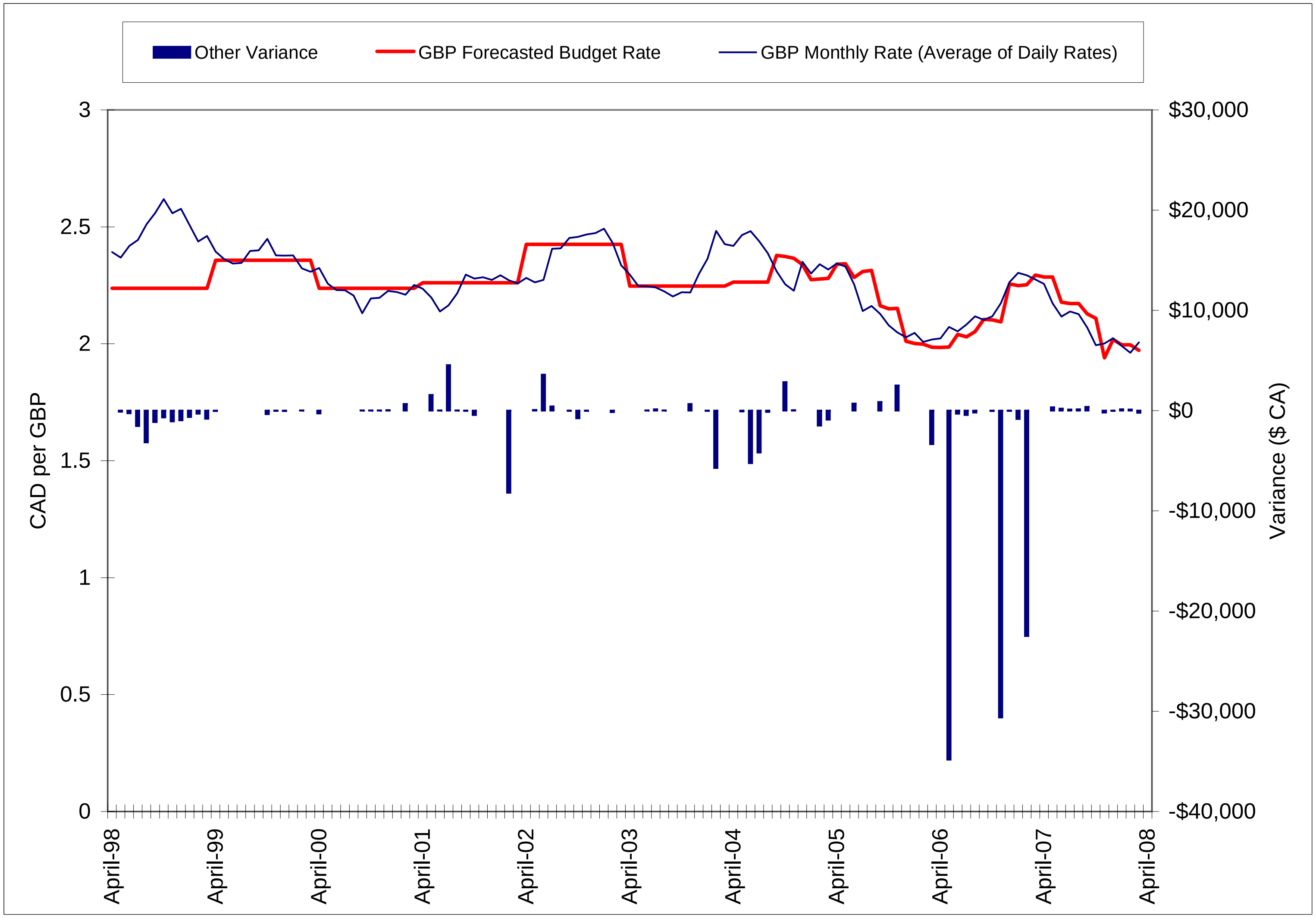

Figure A.3: Rates and Canadian dollar variance on U.K. sterling liquidated obligations (Other category). Left-hand scale shows exchange rate; Right-hand scale shows variance.

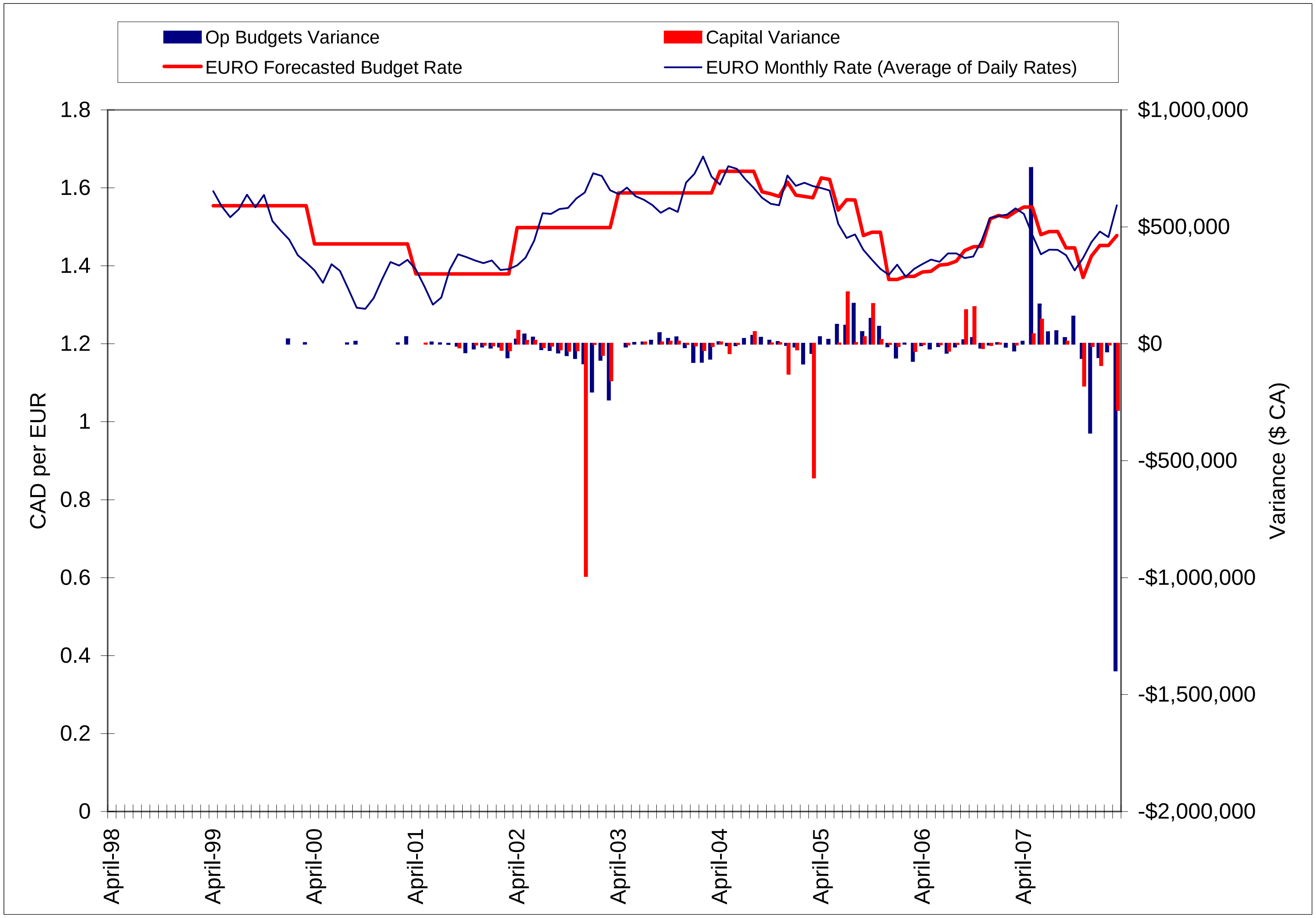

Figure A.4: Rates and Canadian dollar variance on euro-liquidated obligations (Operating Budget and Capital (equipment) categories). Left-hand scale shows exchange rate; Right-hand scale shows variance.

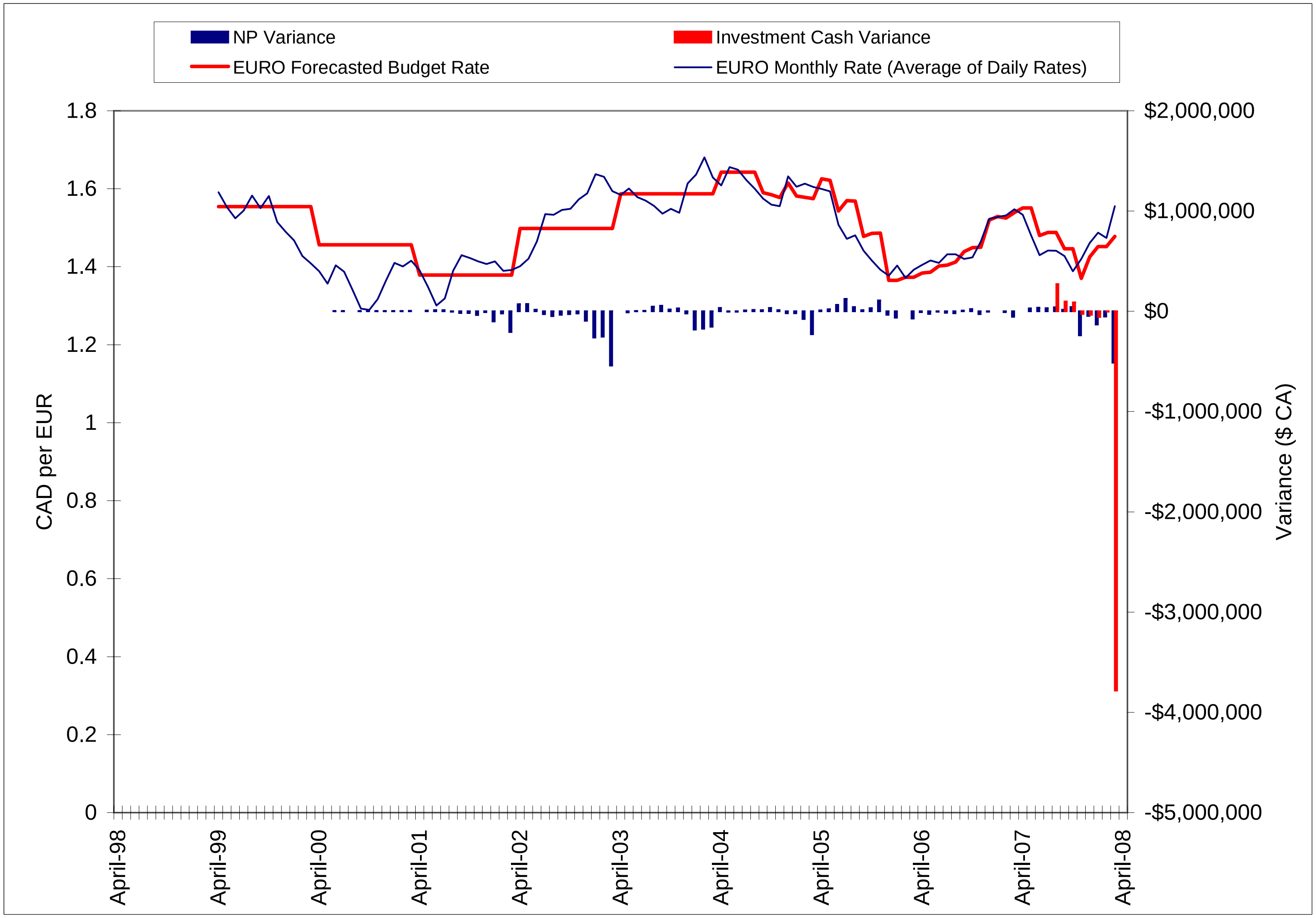

Figure A.5: Rates and Canadian dollar variance on euro liquidated obligations (National Procurement and Investment Cash categories). Left-hand scale shows exchange rate; Right-hand scale shows variance.

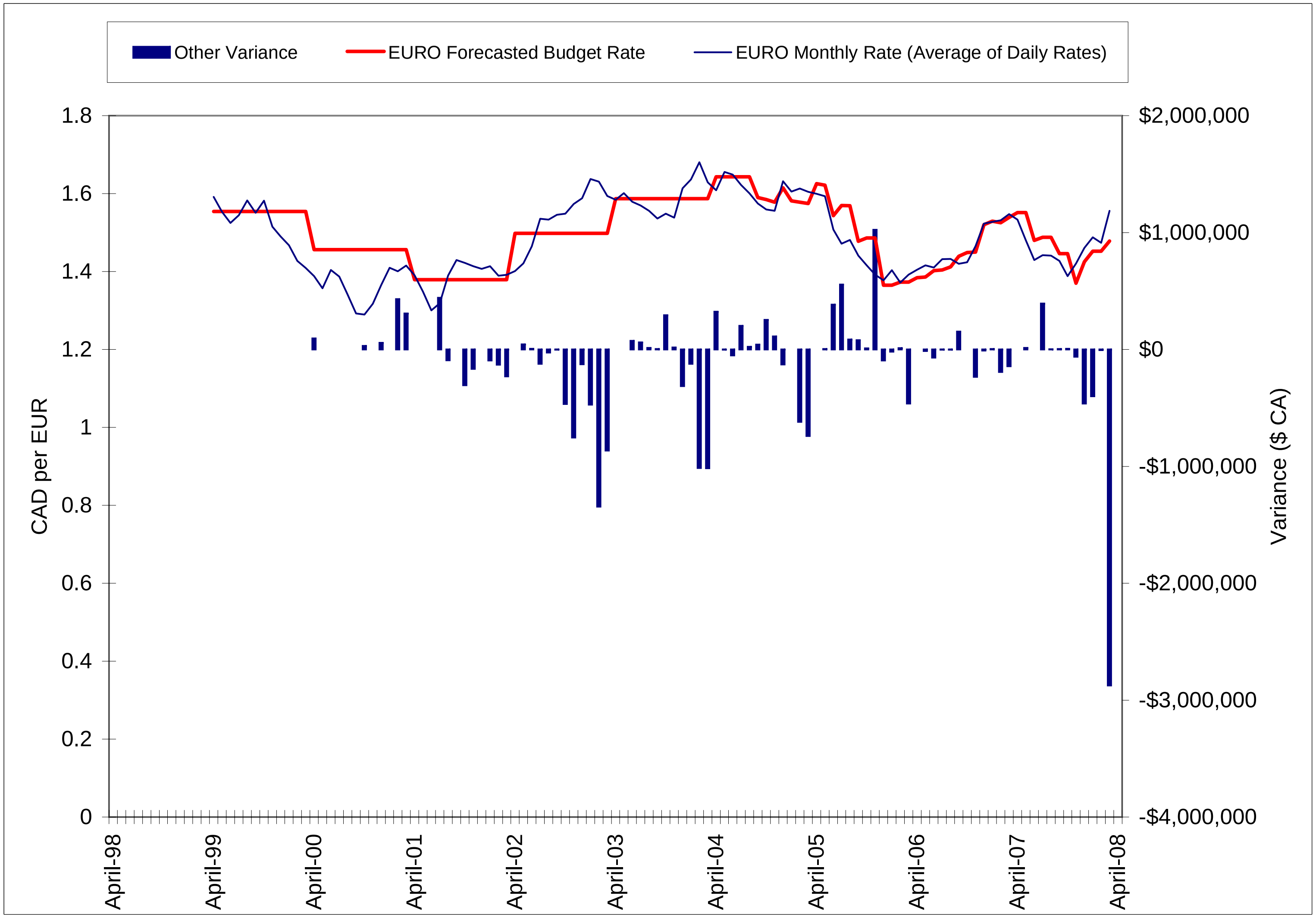

Figure A.6: Rates and Canadian dollar variance on euro liquidated obligations (Other category). Left-hand scale shows exchange rate; Right-hand scale shows variance.

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Annex B: Plots of Actuals, Fit Values and Rescaled Residuals for USD Funds

Table B.1 statistics give some indication about the goodness of fit of the USD models. Except for the investment cash funds, V511, V510 and their roll-up, most funds are well defined by the models. In the case of the small sample size investment cash models, the total variance of the data is so large that the R^2 values become meaningless. For the rescaled residuals, obtained by dividing the residuals by the estimate of the white noise standard deviation, the mean is effectively zero and the variance is one, to support the realization of a white noise sequence.

Table B.1: USD model statistics

Fund	R^2	MSE	Residual Mean
L101	0.915	15.421	-2.698×10^{-5}
L501	0.986	0.879	-7.086×10^{-6}
L518	0.947	0.0579	-8.499×10^{-5}
C503	0.773	243.377	-2.111×10^{-5}
C113	0.908	25.997	-1.374×10^{-5}
V511	N/A	N/A	5.560×10^{-5}
V510	N/A	N/A	-4.796×10^{-4}
C001	0.801	2.496	1.061×10^{-4}
C107	0.857	0.123	1.056×10^{-2}
C160	0.973	0.0764	2.350×10^{-4}
Operational Budgets	0.940	22.094	1.012×10^{-4}
Investment Cash	N/A	N/A	6.762×10^{-4}
Other	0.717	3.859	1.077×10^{-4}

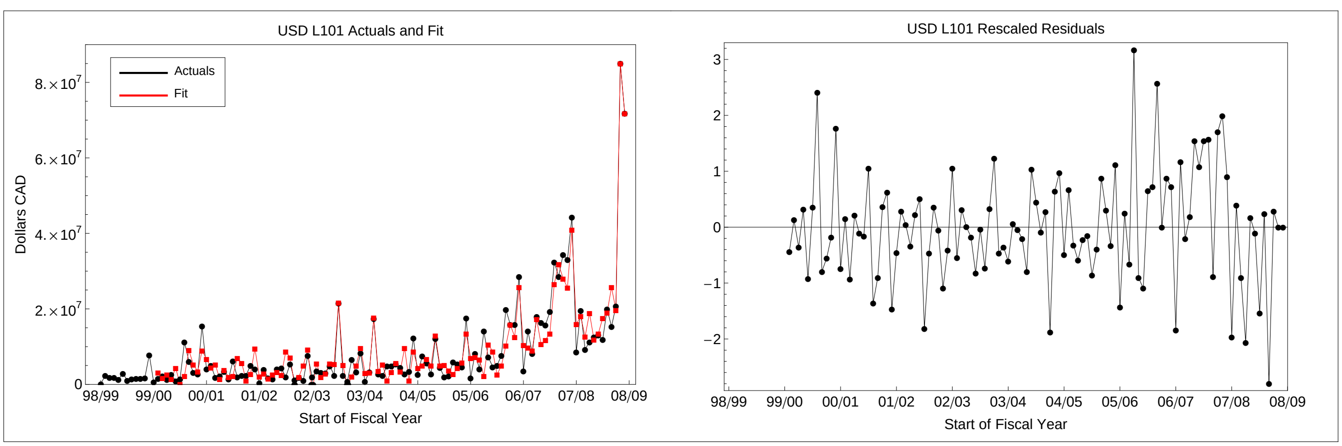

Figure B.1: USD L101 fund actual data, model fit and rescaled residuals

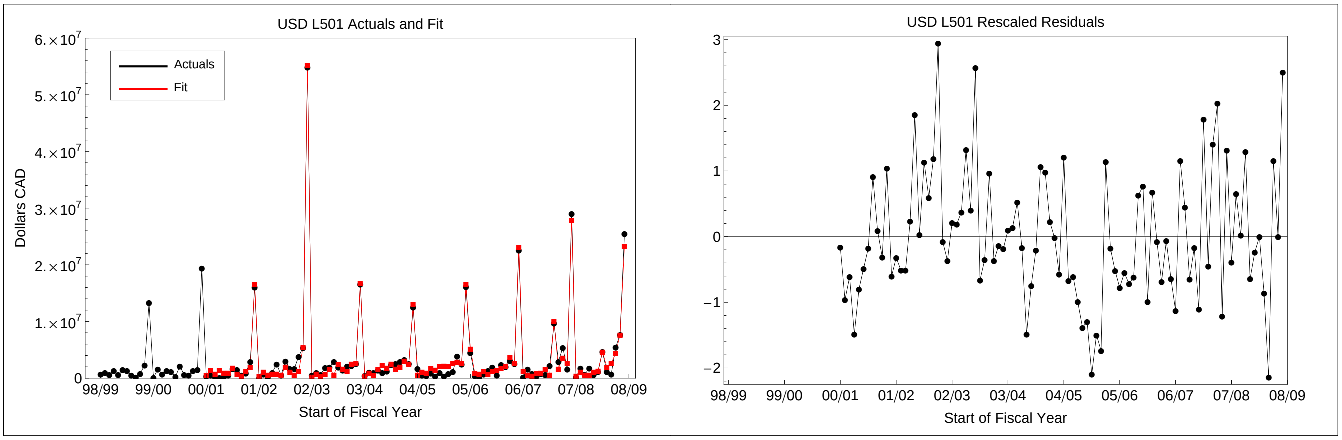

Figure B.2: USD L501 fund actual data, model fit and rescaled residuals

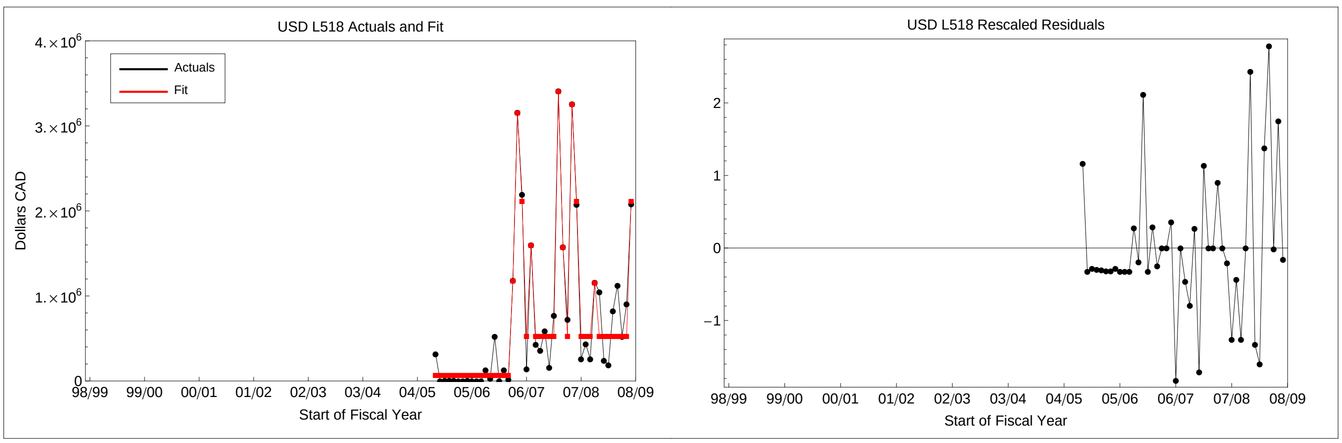

Figure B.3: USD L518 fund actual data, model fit and rescaled residuals

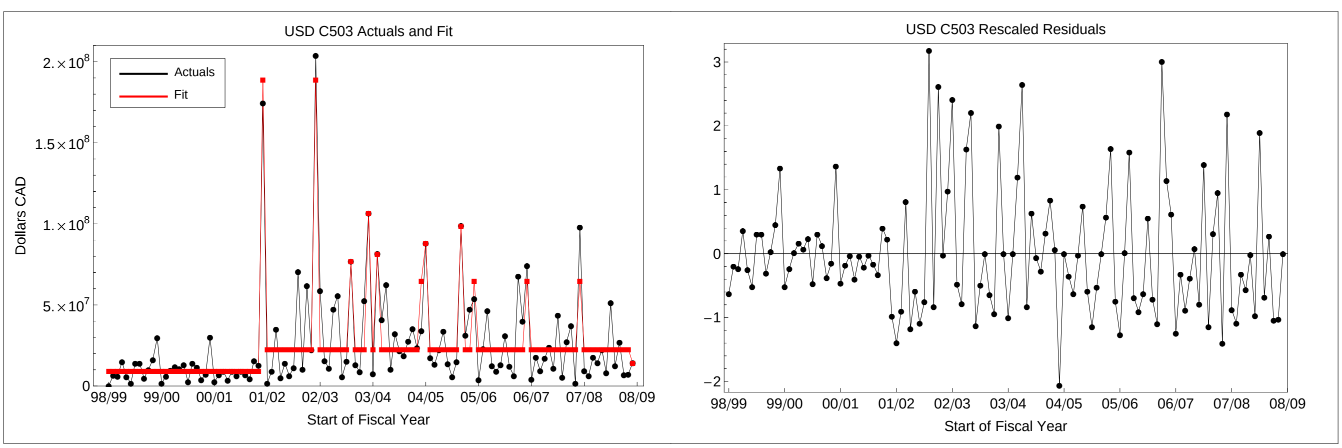

Figure B.4: USD C503 fund actual data, model fit and rescaled residuals

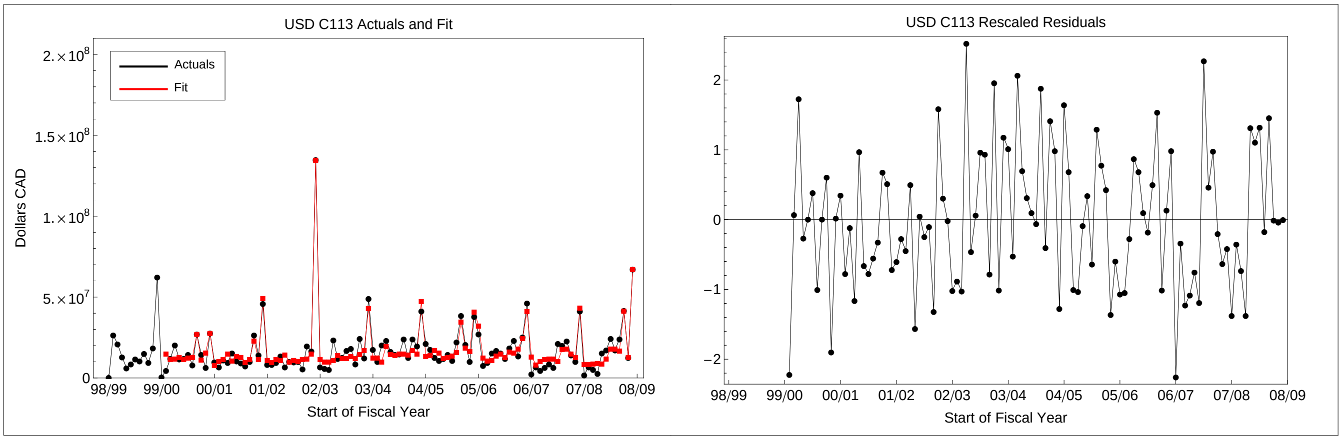

Figure B.5: USD C113 fund actual data, model fit and rescaled residuals

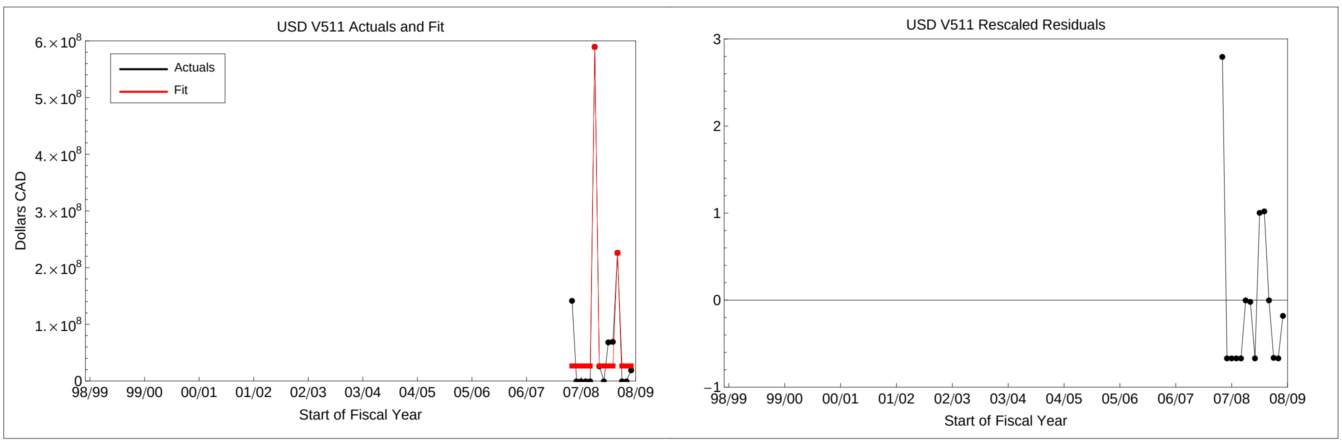

Figure B.6: USD V511 fund actual data, model fit and rescaled residuals

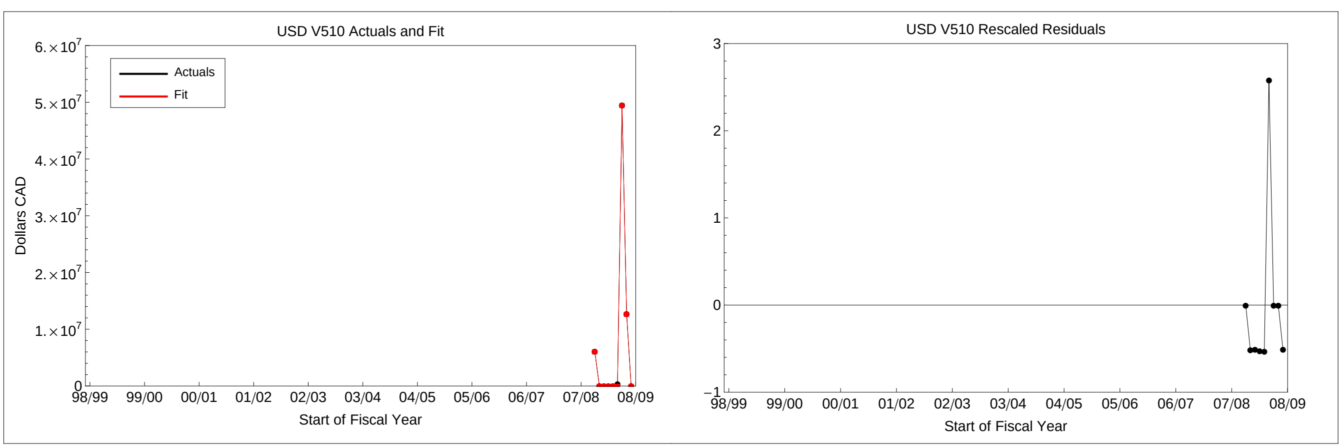

Figure B.7: USD V510 fund actual data, model fit and rescaled residuals

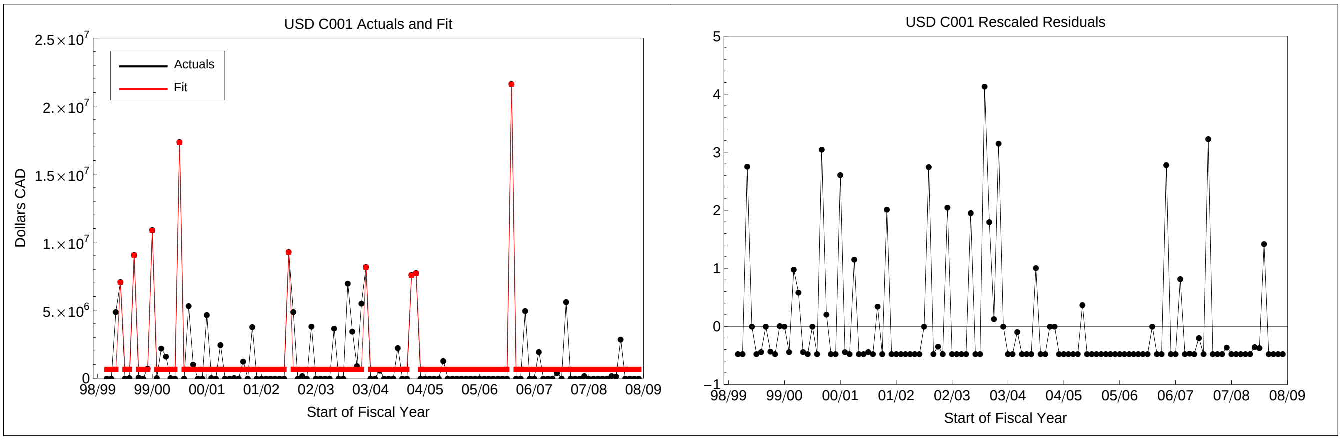

Figure B.8: USD C001 fund actual data, model fit and rescaled residuals

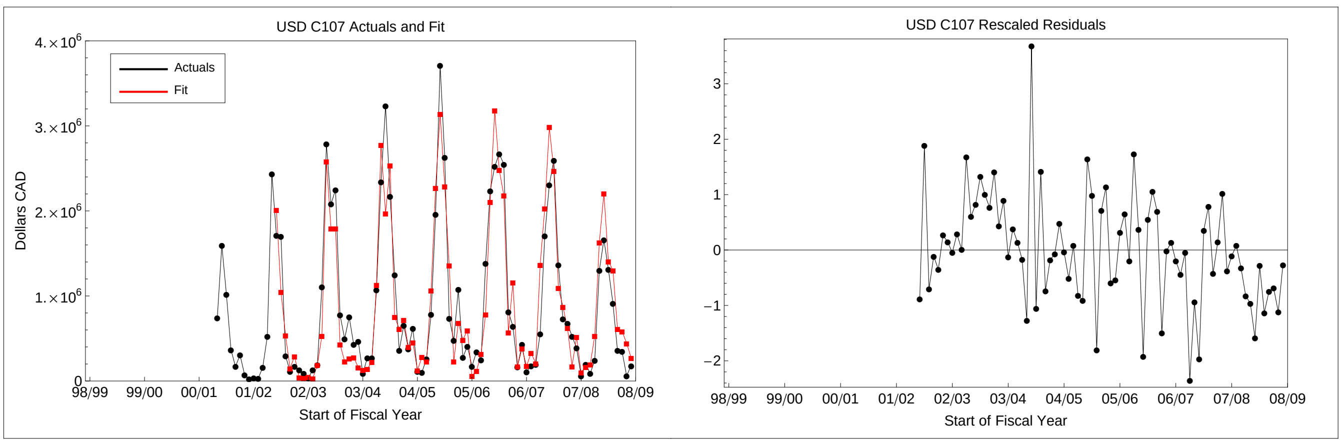

Figure B.9: USD C107 fund actual data, model fit and rescaled residuals

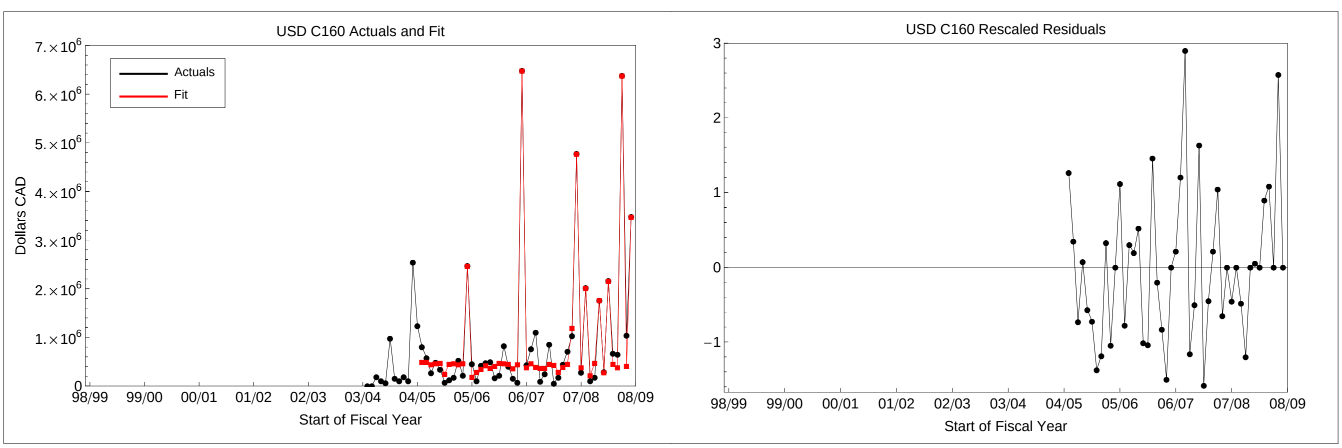

Figure B.10: USD C160 fund actual data, model fit and rescaled residuals

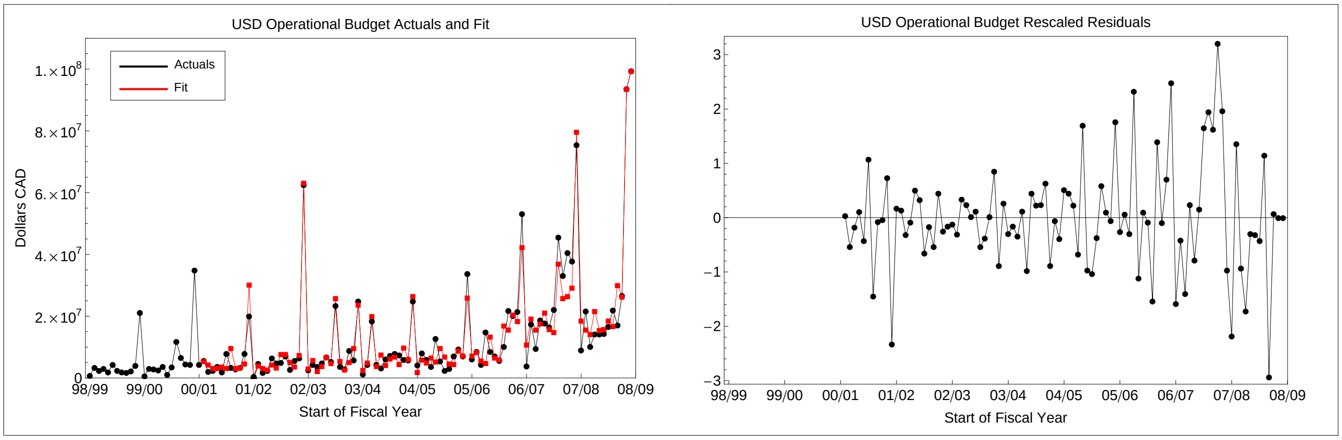

Figure B.11: USD Operational Budgets actual data, model fit and rescaled residuals

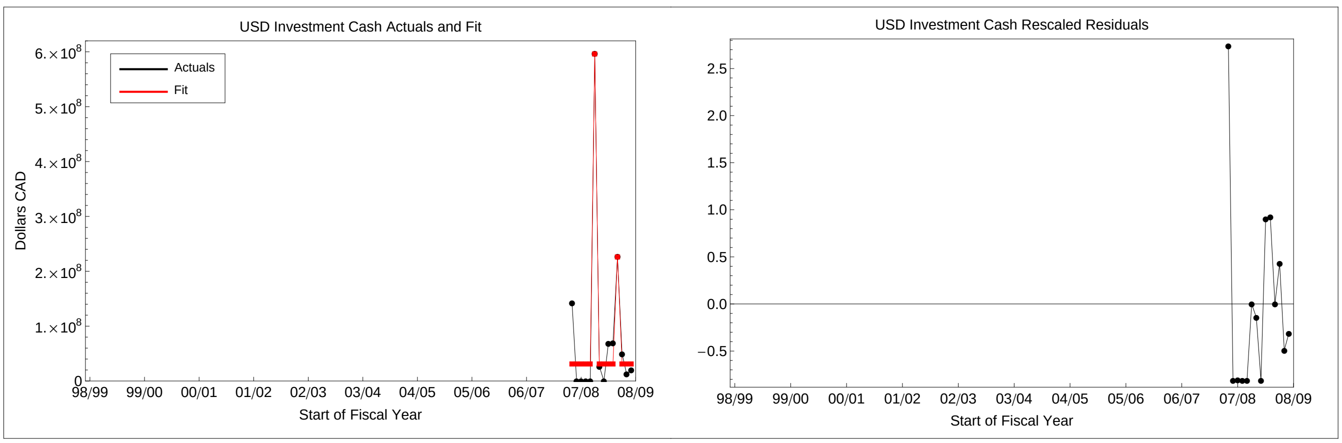

Figure B.12: USD Investment Cash actual data, model fit and rescaled residuals

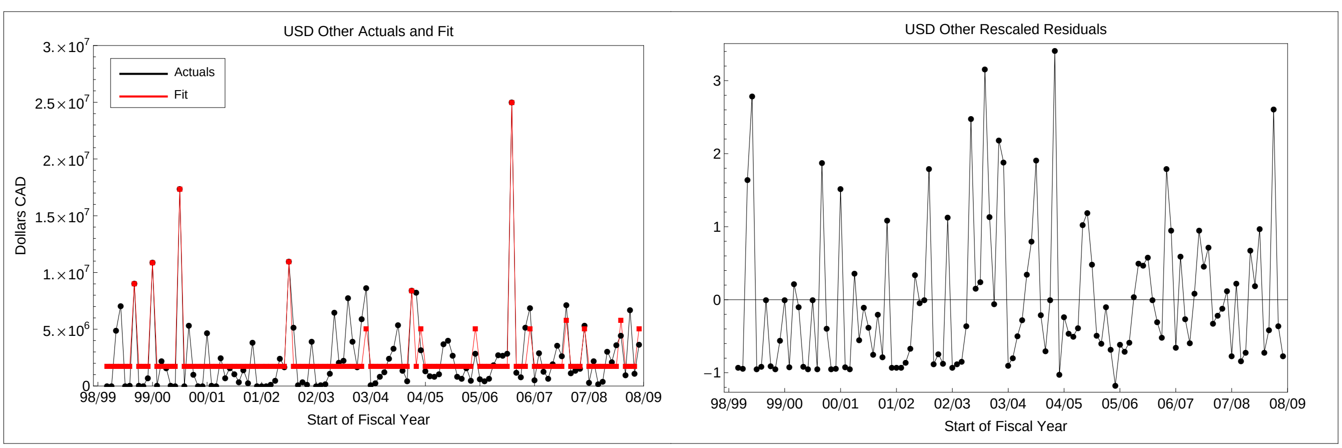

Figure B.13: USD Other funds actual data, model fit and rescaled residuals

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Annex C: Plots of Actuals, Fit Values and Rescaled Residuals for GBP Funds

With the U.S. being Canada's largest trading and defense partner, there is a large difference between annual USD and GBP spending and that is clearly reflected in the quality of the funds data. Of the operational budget funds, GBP L518 is not well defined. Similarly, all GBP investment cash and other funds are characterized by small payments interspersed with large magnitude outliers, leaving Autobox with a challenge to fit the best model possible.

Table C.1 statistics give some indication about the goodness of fit of the GBP models. For GBP L518, it has been stated by DFSC staff that "... for now no changes are expected to occur in GBP and EUR denominated expenditures of this fund, therefore it can be ignored.[18]" Nevertheless, the spending patterns, if any, will be monitored to determine whether or not to drop the fund from further analysis. In the case of V511 and V510, there exists data for both funds but, by 31 March 2008, only V511 had sufficient data to generate a model. The data from both funds, however, were nevertheless combined in the investment cash roll-up.

Except for GBP L518, the rescaled residuals have a mean that is effectively zero and a variance of one, to support the realization of a white noise sequence.

Table C.1: GBP model statistics

Fund	R^2	MSE	Residual Mean
L101	0.721	9.672	-1.357×10^{-4}
L501	0.857	8.266	2.890×10^{-5}
L518	N/A	N/A	-8.640×10^{-1}
C503	0.746	53.950	-4.485×10^{-5}
C113	0.868	295.946	-1.696×10^{-4}
V511	N/A	N/A	1.975×10^{-4}
C001	0.995	3.14×10^{-2}	-2.469×10^{-5}
C107	0.956	6.04×10^{-5}	3.546×10^{-4}
C160	0.996	9.85×10^{-4}	-3.026×10^{-4}
Operational Budgets	0.804	22.889	1.196×10^{-4}
Investment Cash	0.900	4.319	3.874×10^{-4}
Other	0.994	3.58×10^{-2}	5.934×10^{-5}

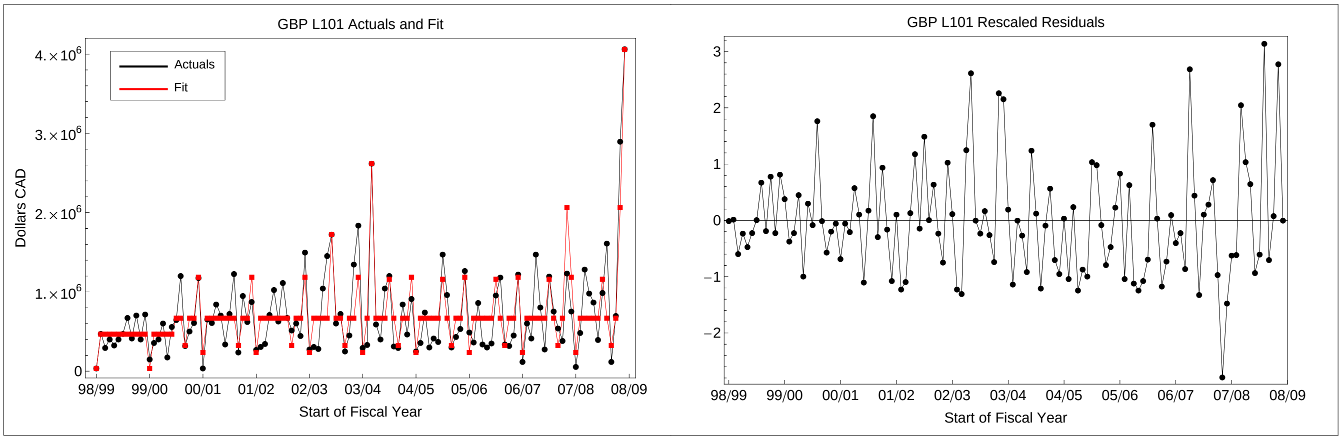

Figure C.1: GBP L101 fund actual data, model fit and rescaled residuals

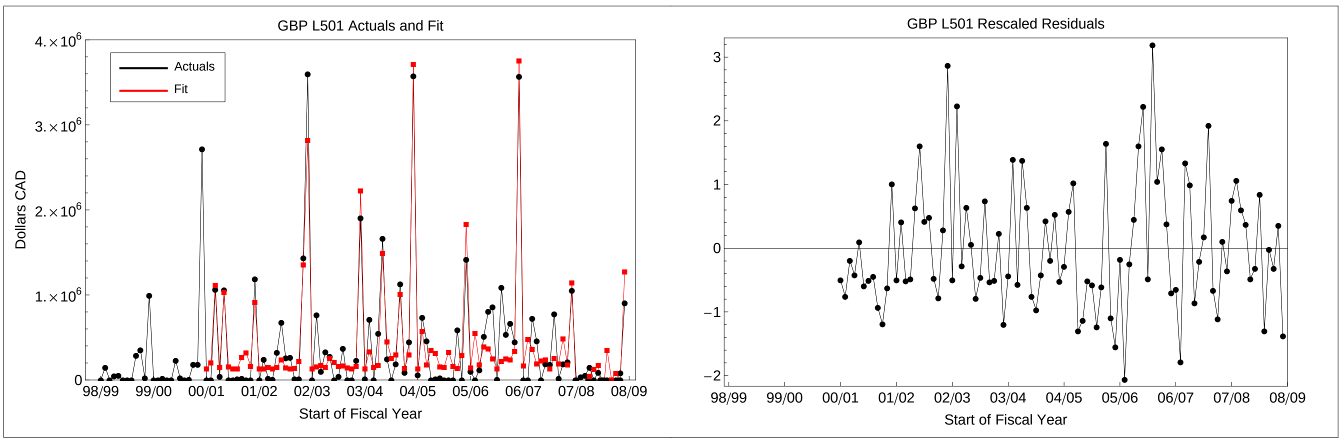

Figure C.2: GBP L501 fund actual data, model fit and rescaled residuals

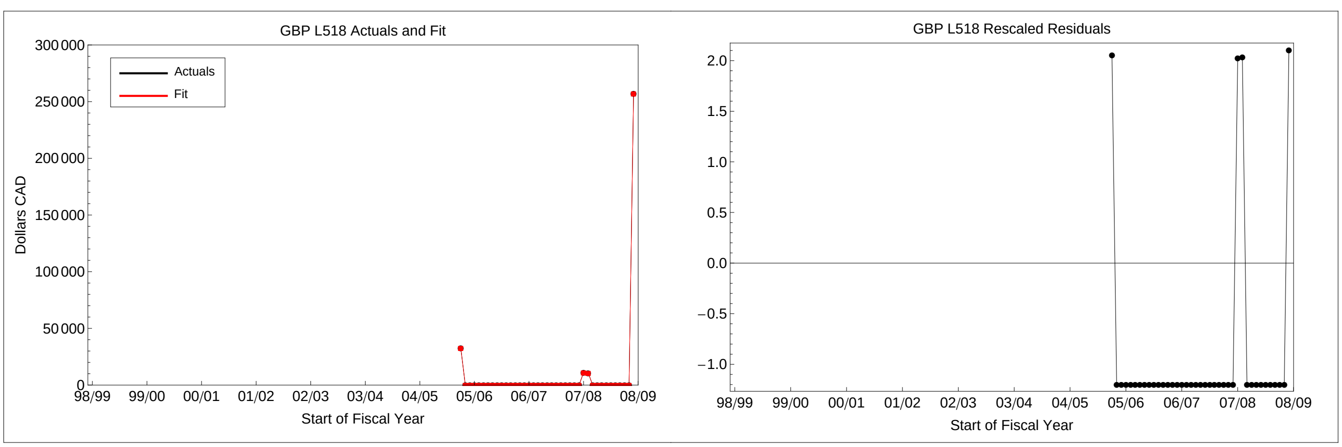

Figure C.3: GBP L518 fund actual data, model fit and rescaled residuals

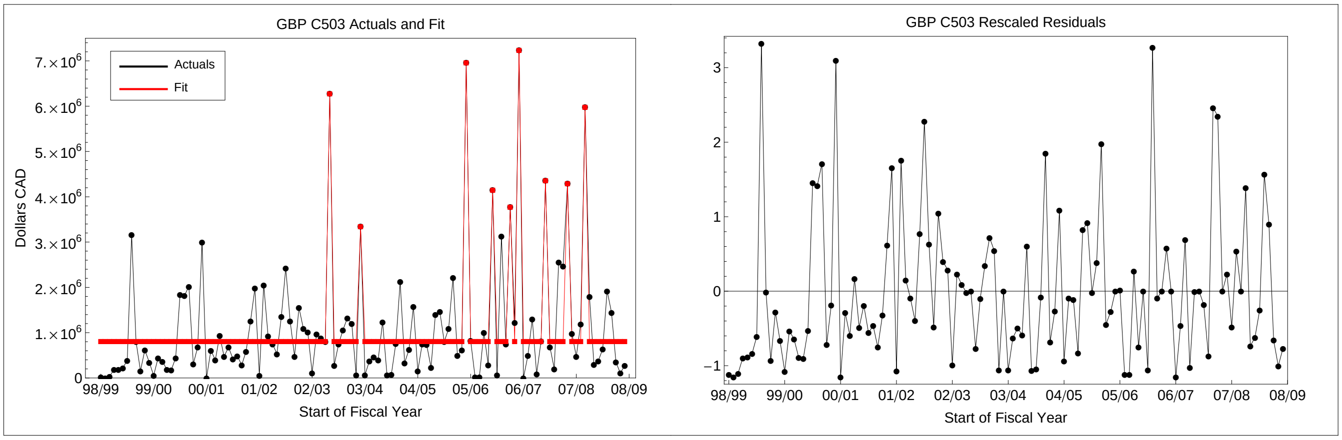

Figure C.4: GBP C503 fund actual data, model fit and rescaled residuals

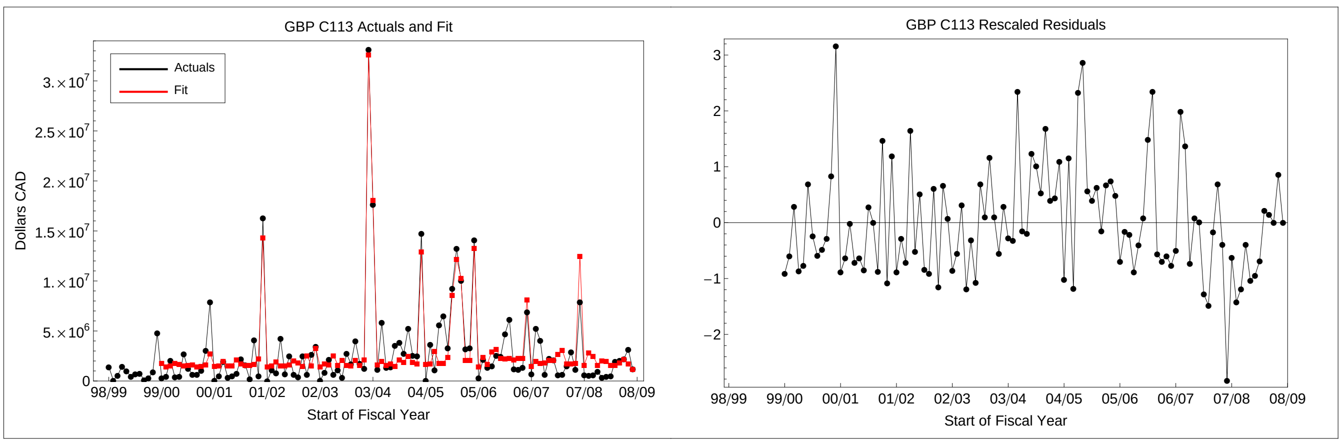

Figure C.5: GBP C113 fund actual data, model fit and rescaled residuals

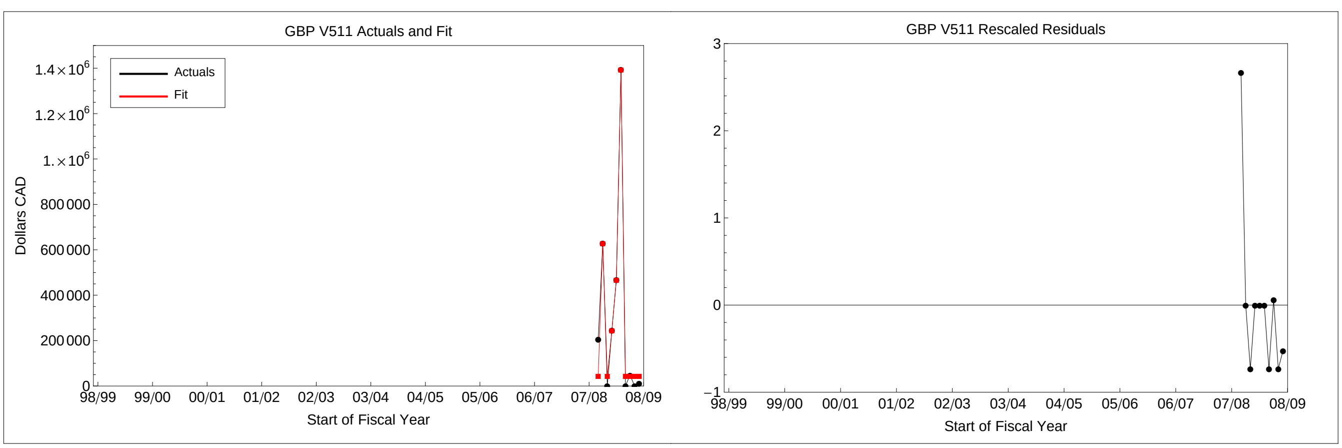

Figure C.6: GBP V511 fund actual data, model fit and rescaled residuals

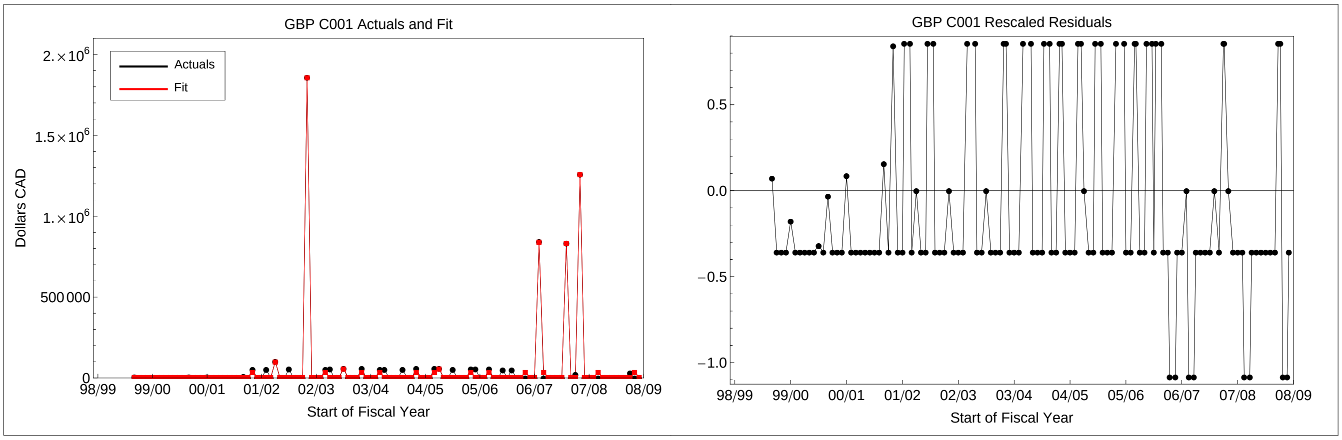

Figure C.7: GBP C001 fund actual data, model fit and rescaled residuals

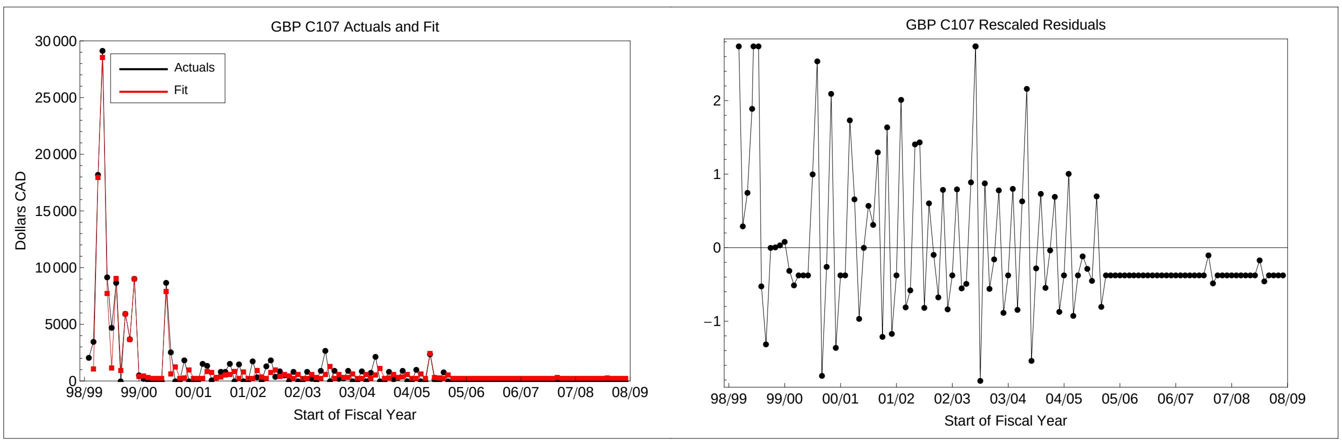

Figure C.8: GBP C107 fund actual data, model fit and rescaled residuals

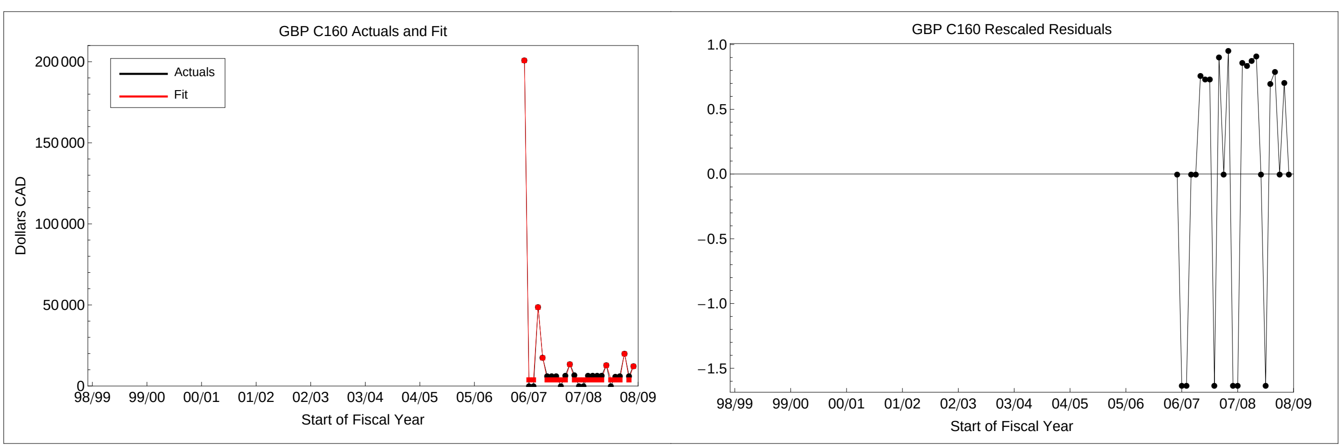

Figure C.9: GBP C160 fund actual data, model fit and rescaled residuals

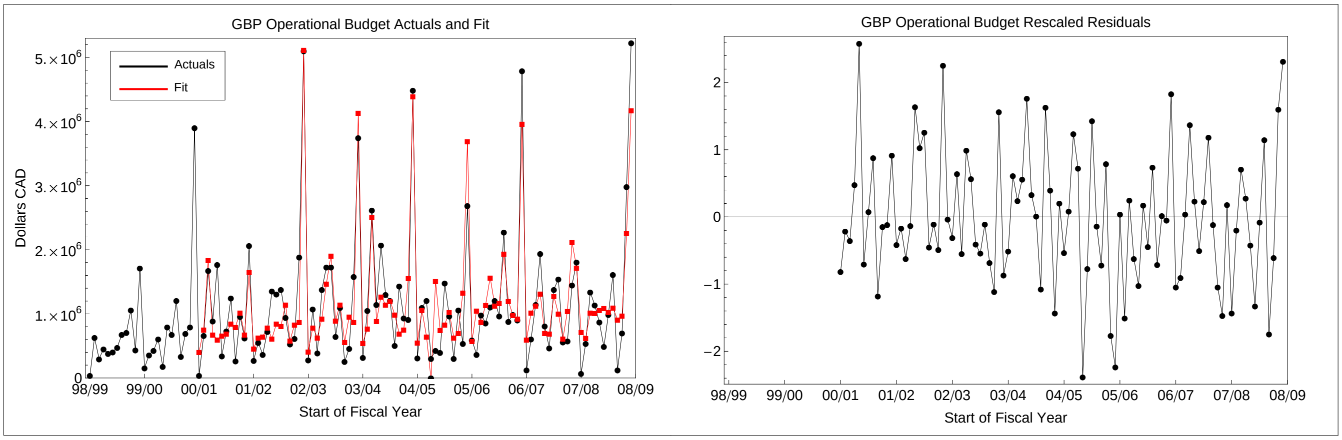

Figure C.10: GBP Operational Budgets actual data, model fit and rescaled residuals

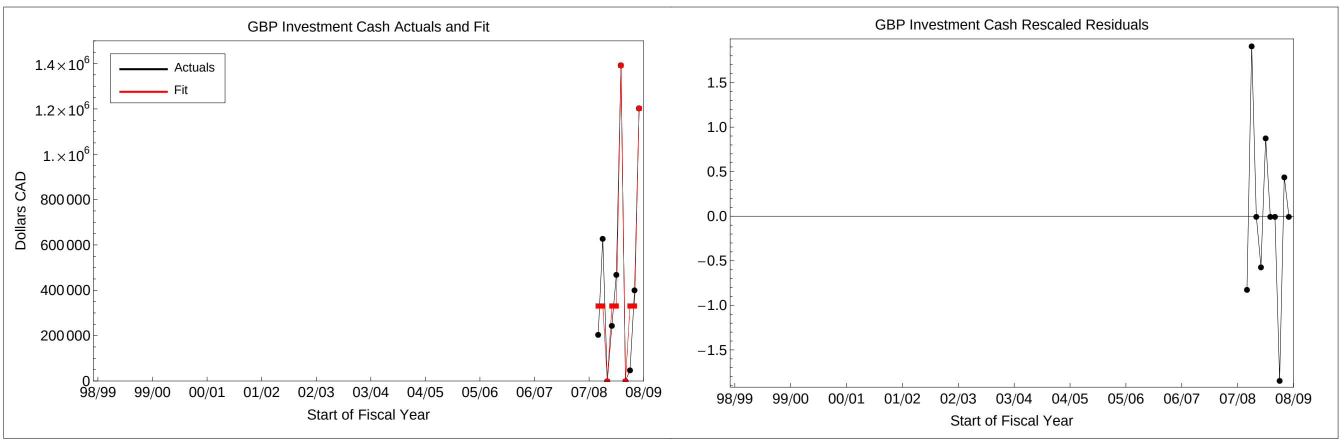

Figure C.11: GBP Investment Cash actual data, model fit and rescaled residuals

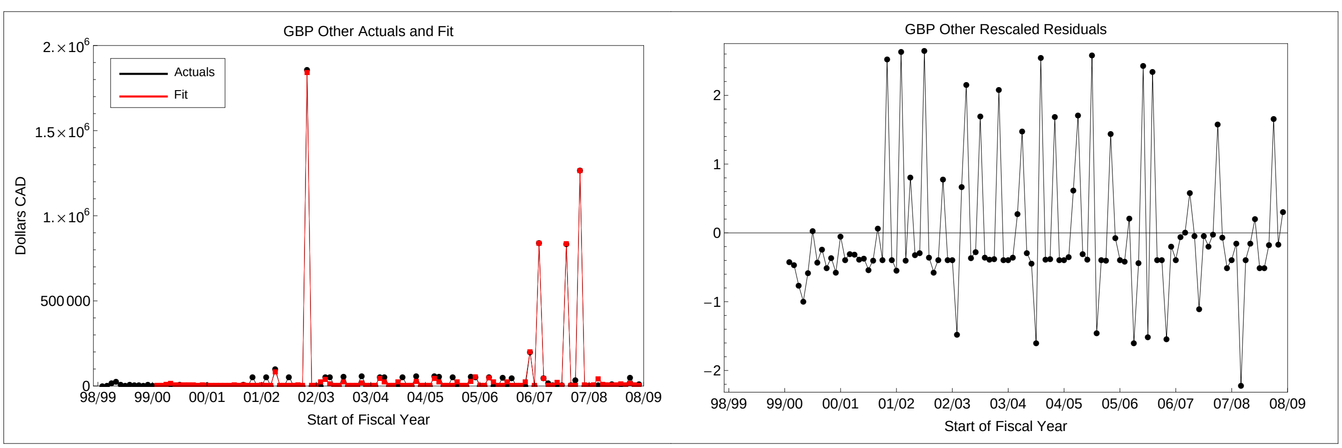

Figure C.12: GBP Other funds actual data, model fit and rescaled residuals

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Annex D: Plots of Actuals, Fit Values and Rescaled Residuals for EUR Funds

Similarly to the GBP funds, EUR funds L518, V510 and C160 are not well defined and Table D.1 statistics give some indication about the goodness of fit of the remaining EUR models. In the case of V511 and V510, there exists data for both funds but, by 31 March 2008, only V510 had sufficient data to generate a model. The data from both funds, however, were nevertheless combined in the investment cash roll-up.

The rescaled residuals of all funds have a mean that is effectively zero and a variance of one, to support the realization of a white noise sequence.

Table D.1: EUR model statistics

Fund	R^2	MSE	Residual Mean
L101	0.966	0.455	3.075×10^{-4}
L501	0.922	1.24×10^{-2}	-1.198×10^{-4}
L518	N/A	N/A	8.060×10^{-4}
C503	0.960	0.887	3.430×10^{-5}
C113	0.943	0.700	1.628×10^{-4}
V510	N/A	N/A	-3.692×10^{-4}
C001	0.816	29.757	2.213×10^{-4}
C107	0.829	1.57×10^{-4}	1.104×10^{-7}
C160	N/A	N/A	-1.425×10^{-4}
Operational Budgets	0.944	0.783	4.098×10^{-5}
Investment Cash	N/A	N/A	-3.982×10^{-5}
Other	0.816	29.780	1.207×10^{-4}

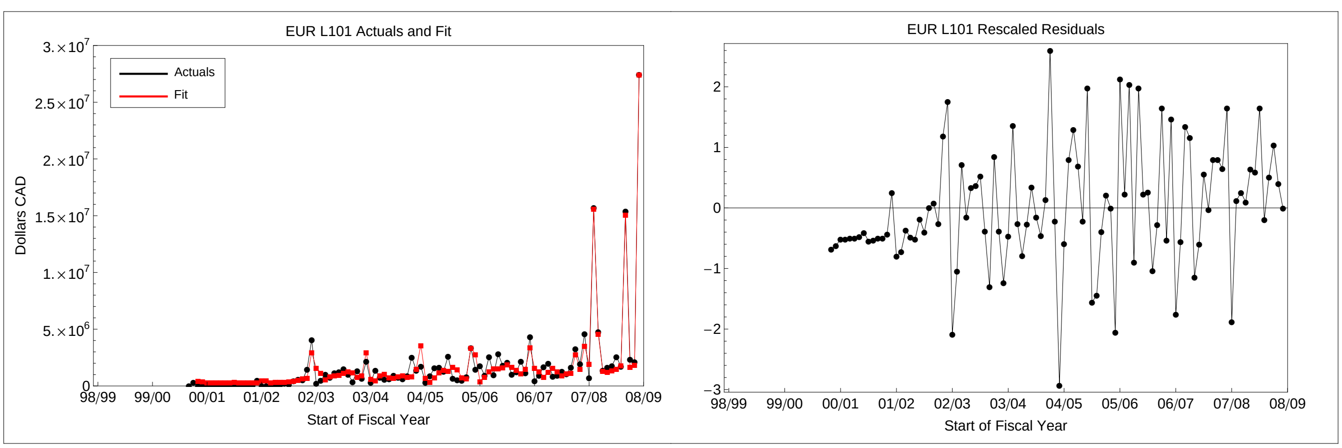

Figure D.1: EUR L101 fund actual data, model fit and rescaled residuals

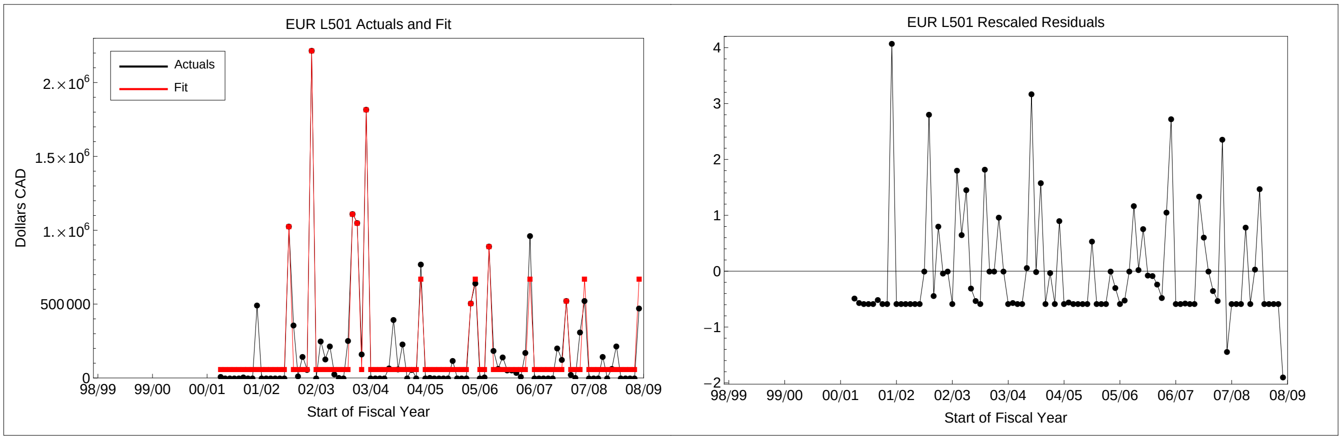

Figure D.2: EUR L501 fund actual data, model fit and rescaled residuals

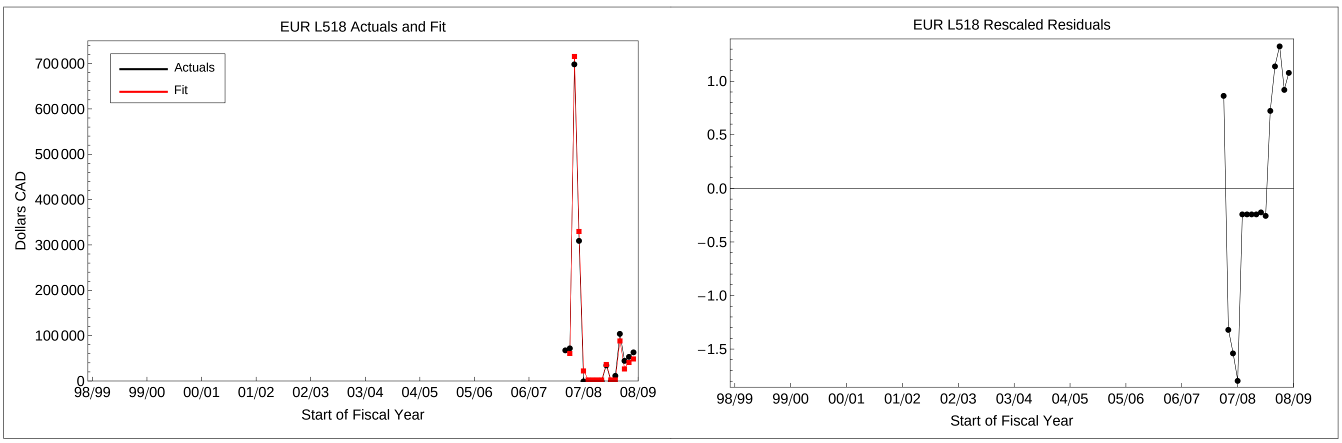

Figure D.3: EUR L518 fund actual data, model fit and rescaled residuals

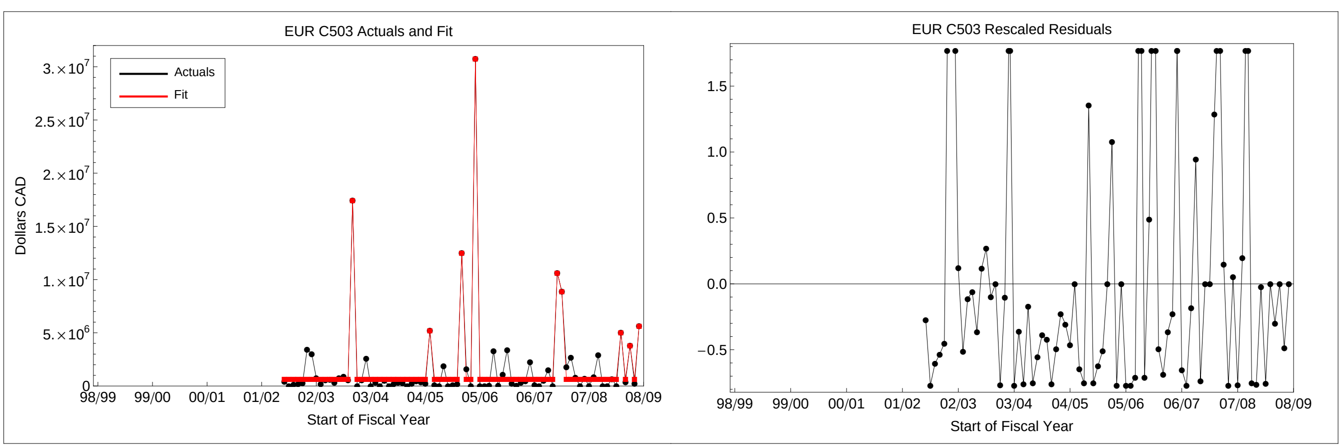

Figure D.4: EUR C503 fund actual data, model fit and rescaled residuals

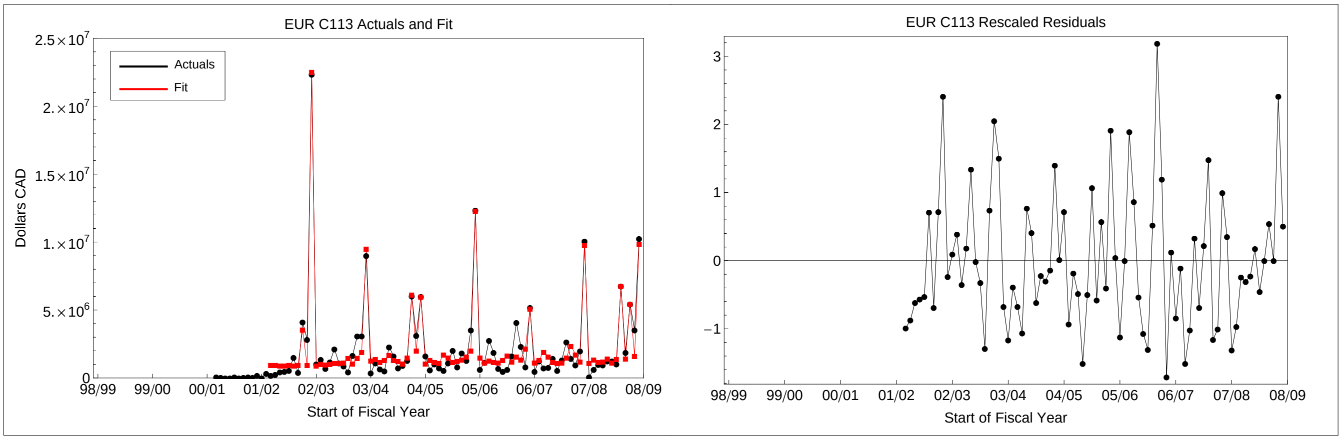

Figure D.5: EUR C113 fund actual data, model fit and rescaled residuals

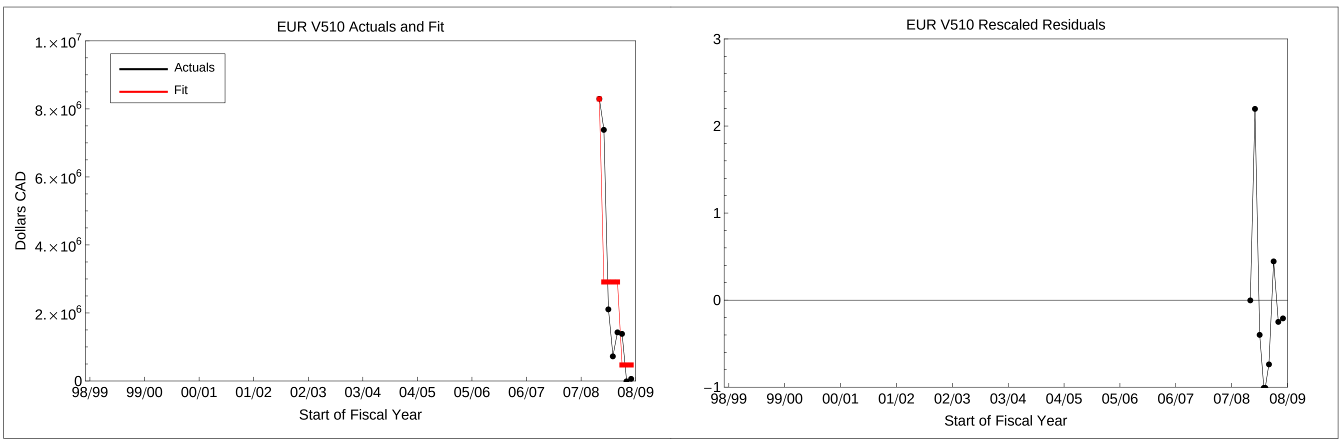

Figure D.6: EUR V510 fund actual data, model fit and rescaled residuals

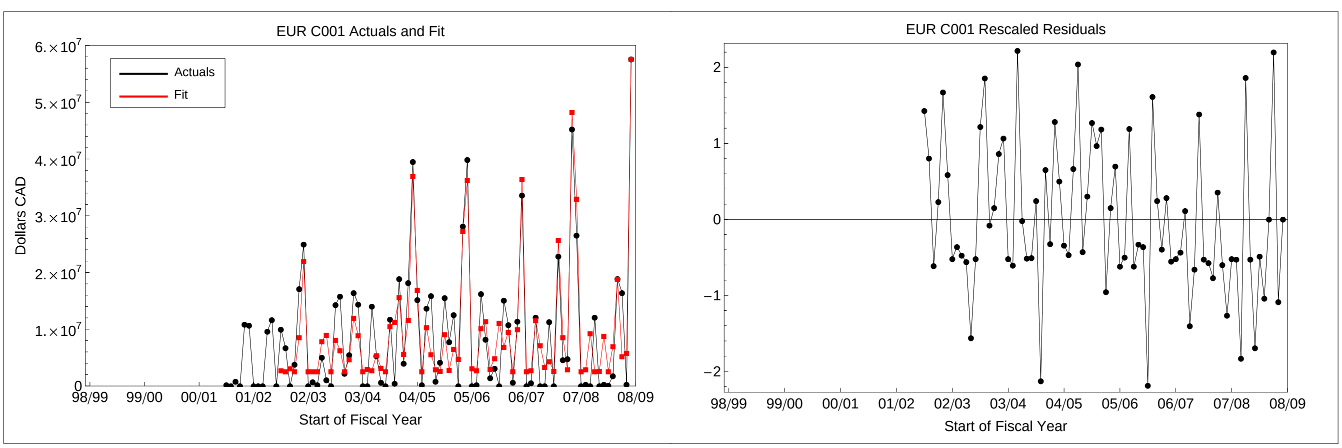

Figure D.7: EUR C001 fund actual data, model fit and rescaled residuals

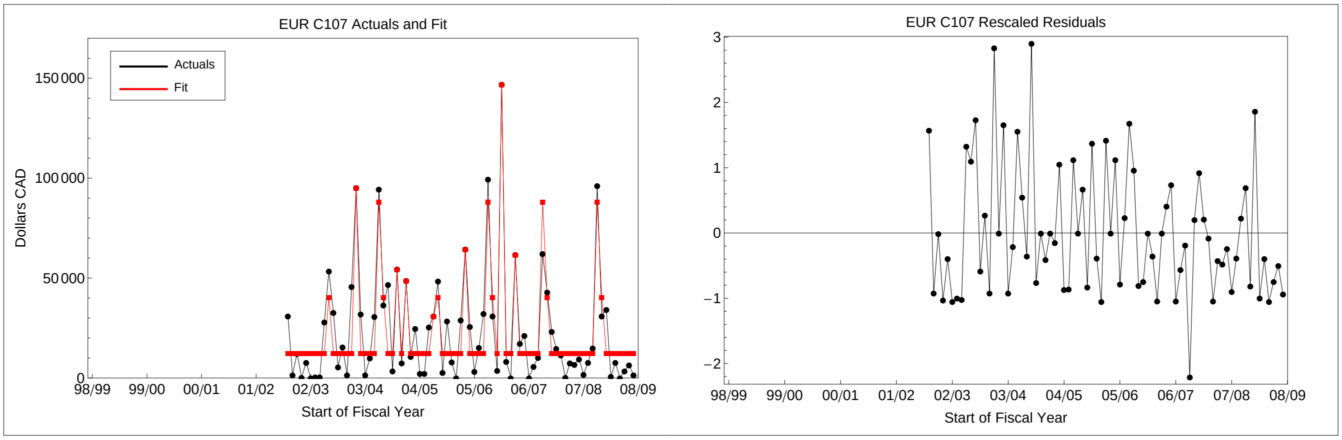

Figure D.8: EUR C107 fund actual data, model fit and rescaled residuals

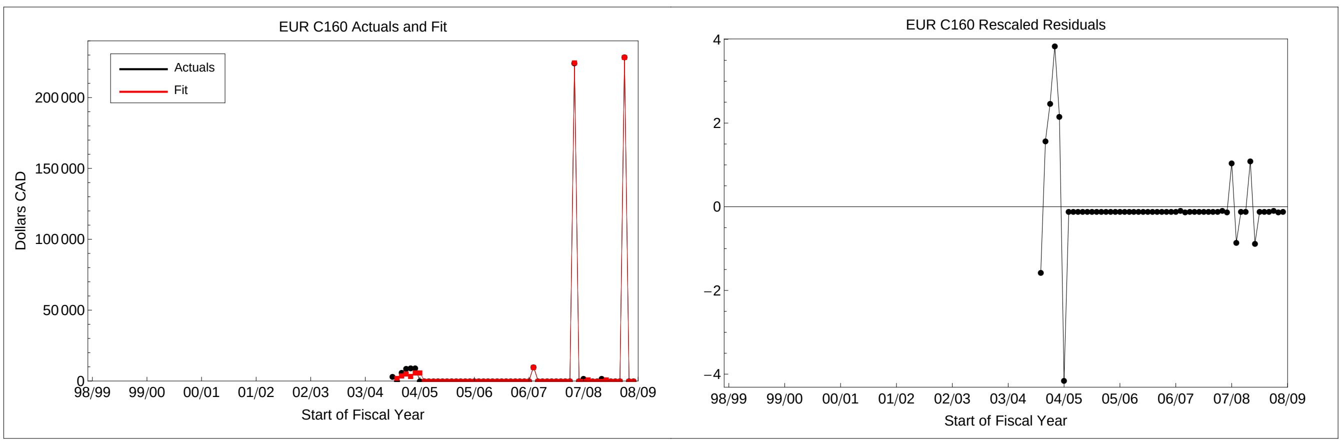

Figure D.9: EUR C160 fund actual data, model fit and rescaled residuals

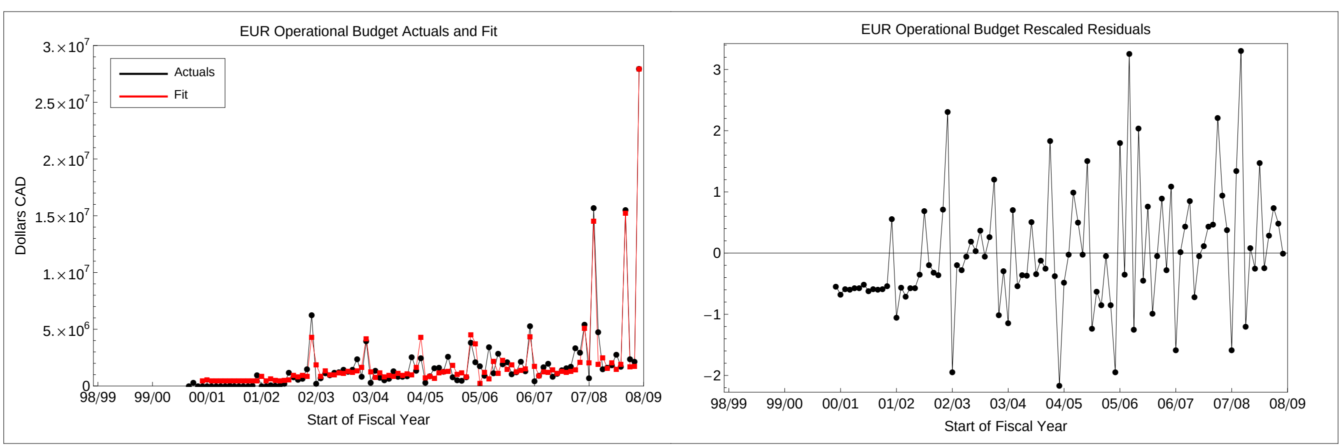

Figure D.10: EUR Operational Budgets actual data, model fit and rescaled residuals

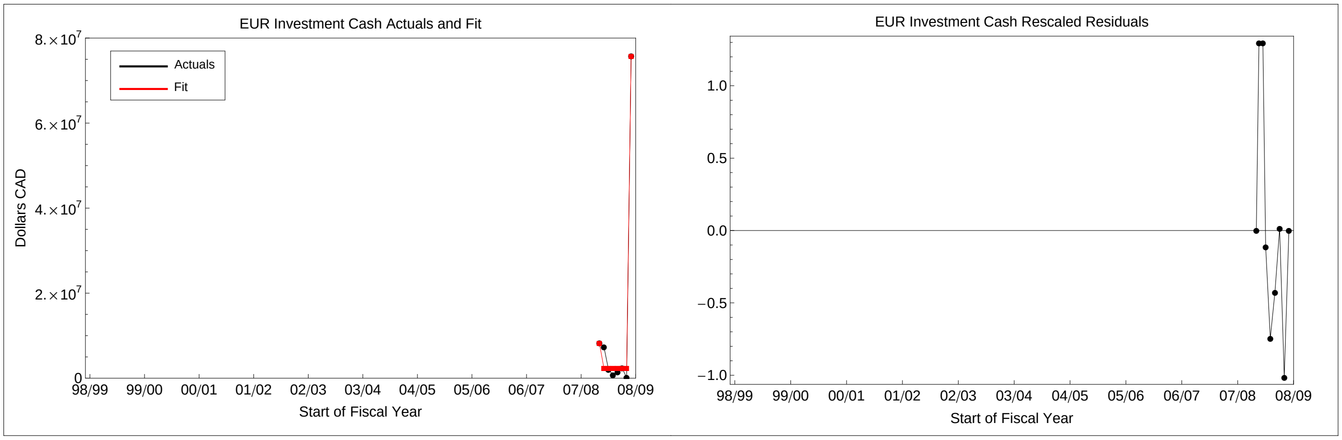

Figure D.11: EUR Investment Cash actual data, model fit and rescaled residuals

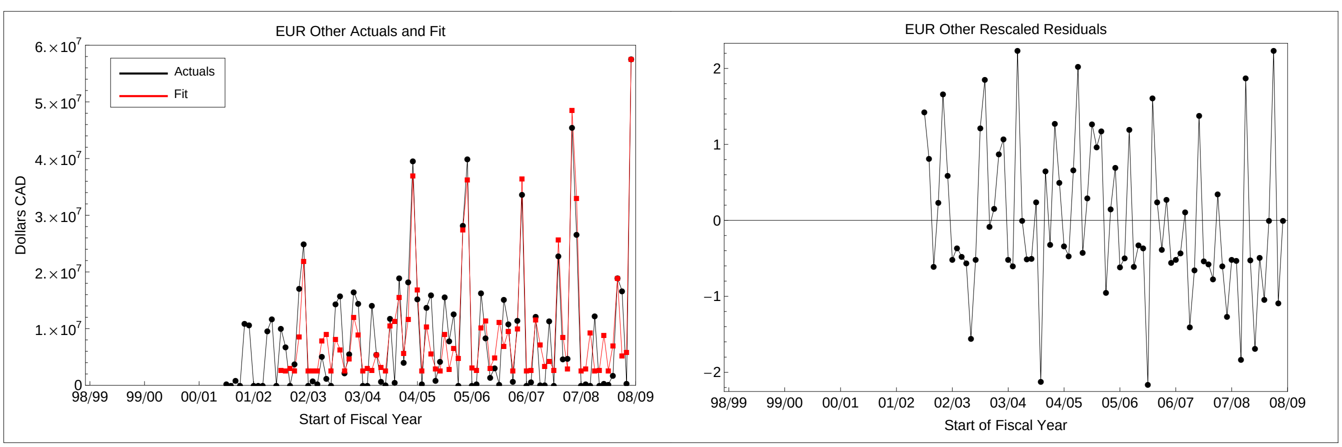

Figure D.12: EUR Other funds actual data, model fit and rescaled residuals

List of Acronyms

ACF	Autocorrelation Function
ADM(Fin CS)	Assistant Deputy Minister (Finance and Corporate Services)
ADM(Mat)	Assistant Deputy Minister (Materiel)
AR	Autoregressive
ARIMA	Autoregressive Integrated Moving Average
Autobox	Automatic Box-Jenkins
BFY	Budget Fiscal Year
CAD	Canadian Dollar
CC	Capability Component
CCTR	Cost Centre
CFE	Cumulative Sum of Forecast Errors
CK	Currency Type
DFPPC	Director Force Planning and Programme Coordination
DIN	Defence Information Network
DMG Compt	Director Materiel Group Comptroller
DMGOR	Director Material Group Operational Research
DND	Department of National Defence
DSFC	Director Strategic Finance and Costing
DSP	Defence Service Program
DT	Document Type
ET	Eastern Standard Time
EUR	Euro
FCTR	Fund Centre
FHS	Filtered Historical Simulation
FMAS	Financial and Managerial Accounting Systems
FOREX	FOReign EXchange
FP	Financial Period
FRNAMT	Foreign Amount
GARCH	Generalized Autoregressive Conditional Heteroskedasticity
GBP	U.K. Pound Sterling
GDP	Gross Domestic Product
GL	General Ledger
i.i.d.	Independent and Identically Distributed
IM	Information Management
IT	Information Technology
KR	Vendor Invoice (German)
MA	Moving Average
MAD	Mean Absolute Deviation

MAPE	Mean Absolute Percentage Error
MLE	Maximum Likelihood Estimation
MSE	Mean Squared Error
NP	National Procurement
Perc.	Percentile
PPP	Purchasing Power Parity
QQ	Quantile-Quantile
RMSE	Root Mean Squared Error
SAS	Statistical Analysis Software
SPSS	Statistical Package for the Social Sciences
USD	U.S. Dollars
VaR	Value at Risk

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In January 2007, the theory and application of the FOREX (FOReign EXchange) risk assessment model was developed and applied to the Assistant Deputy Minister (Materiel) (ADM(Mat)) National Procurement and Capital (equipment) accounts to forecast the worse-case loss in expenditures at a specific confidence level over a certain period of time due to the volatility in foreign currency transactions.

With the success of the original FOREX model, the Assistant Deputy Minister (Finance and Corporate Services) has a requirement to expand the model to include the original two ADM(Mat) accounts, national procurement and capital (equipment), plus eight additional funds that each account for over $10M in foreign currency transactions every year. Unlike the manual approach used in the original study, this study uses the Autobox (Automated Box-Jenkins) application to forecast fund expenditures, while GARCH (Generalized Autoregressive Conditional Heteroskedasticity) models are built to forecast the time-varying volatilities of foreign currency returns. These diverse methodologies are then combined into an overall departmental Value-at-Risk model to determine the maximum expected loss from adverse exchange rate fluctuations over the budget year.

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